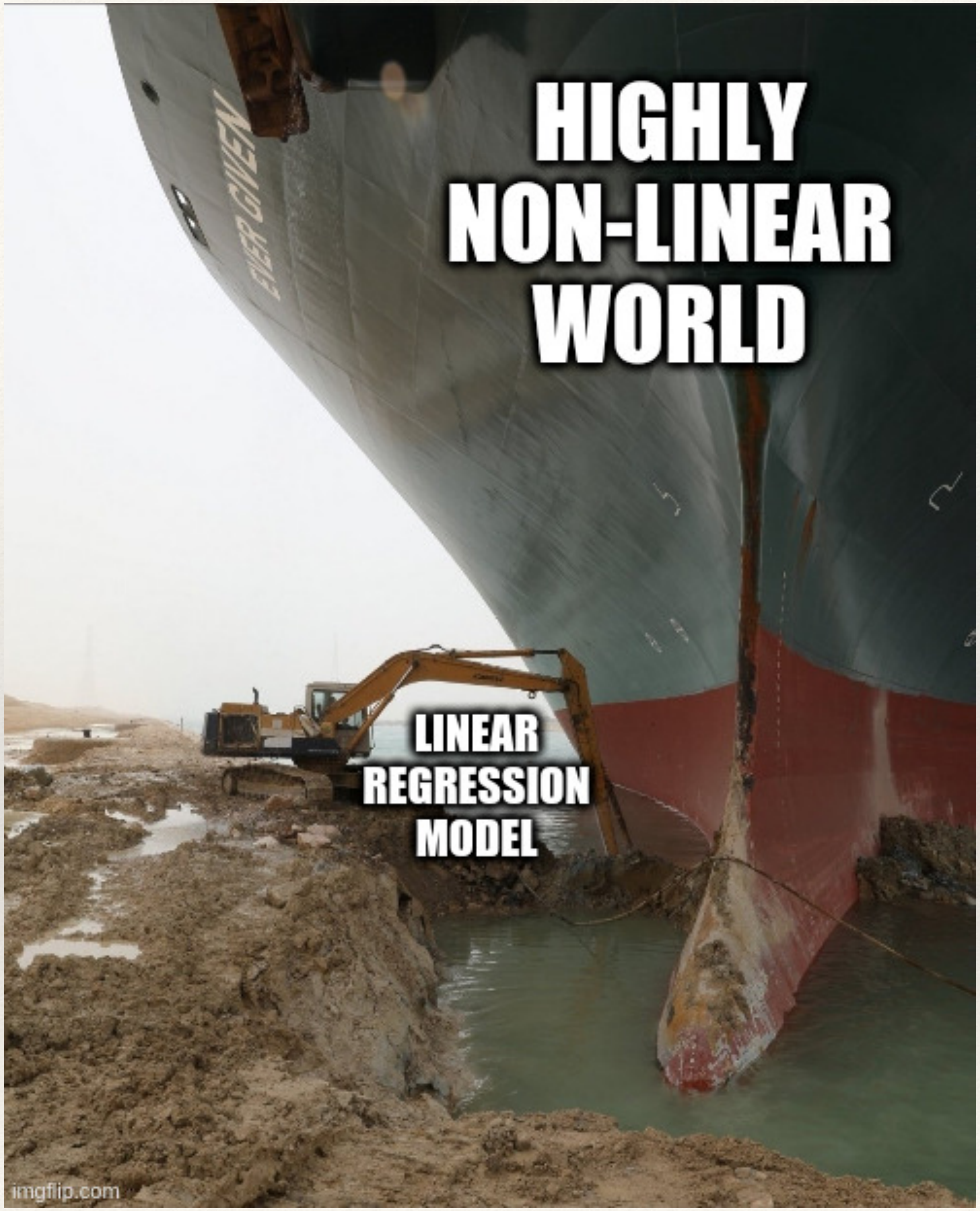
Interactions

Introduction to Statistics



The Plan for Today

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- **Heterogeneous Treatment Effects**

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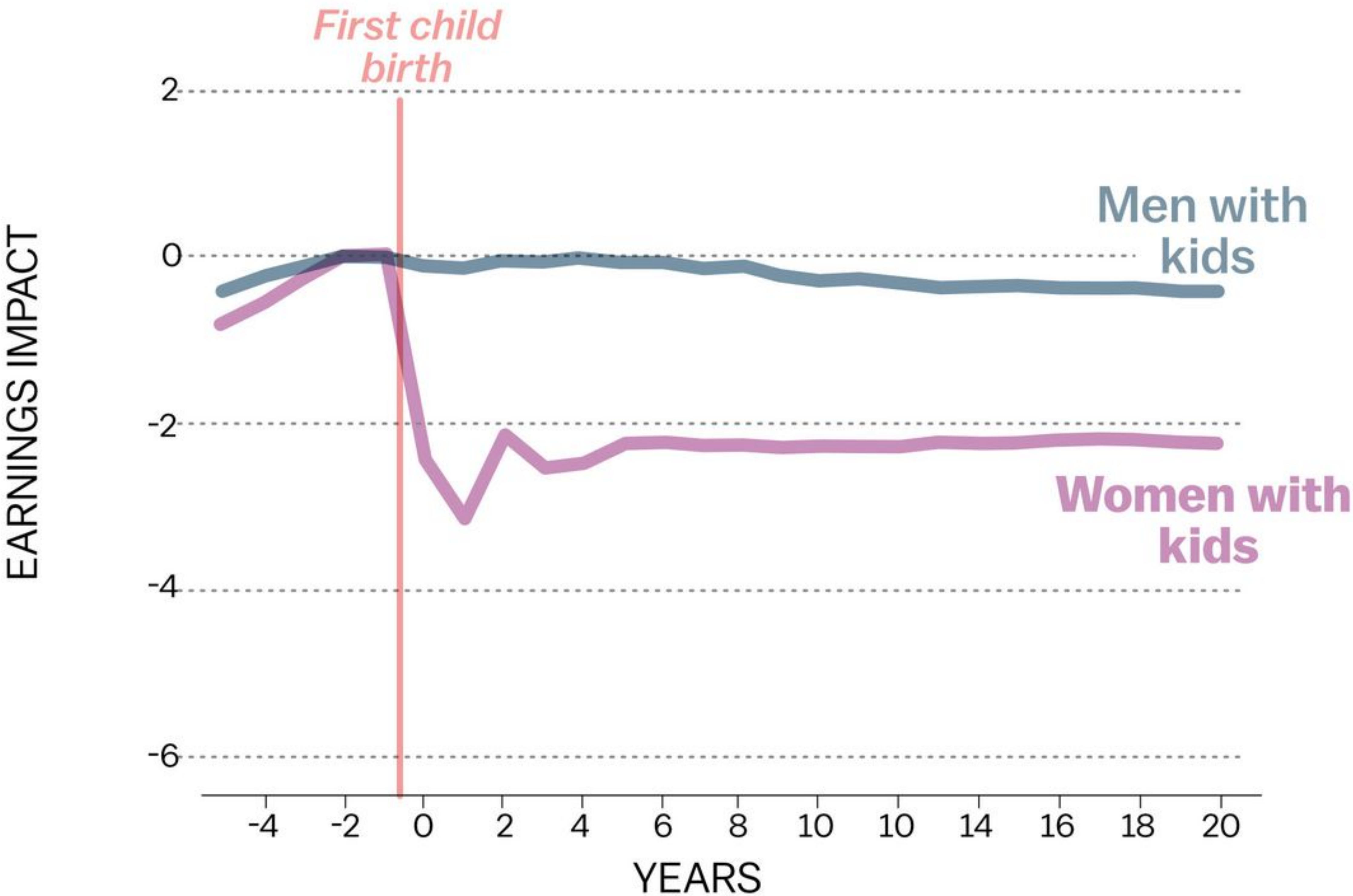
  - Intuition: what's the effect of parenthood on earnings? Well, *depends*.

Women's earnings drop significantly after having a child. Men's don't.



Source: "Children and gender inequality: Evidence from Denmark," National Bureau of Economic Research

Vox

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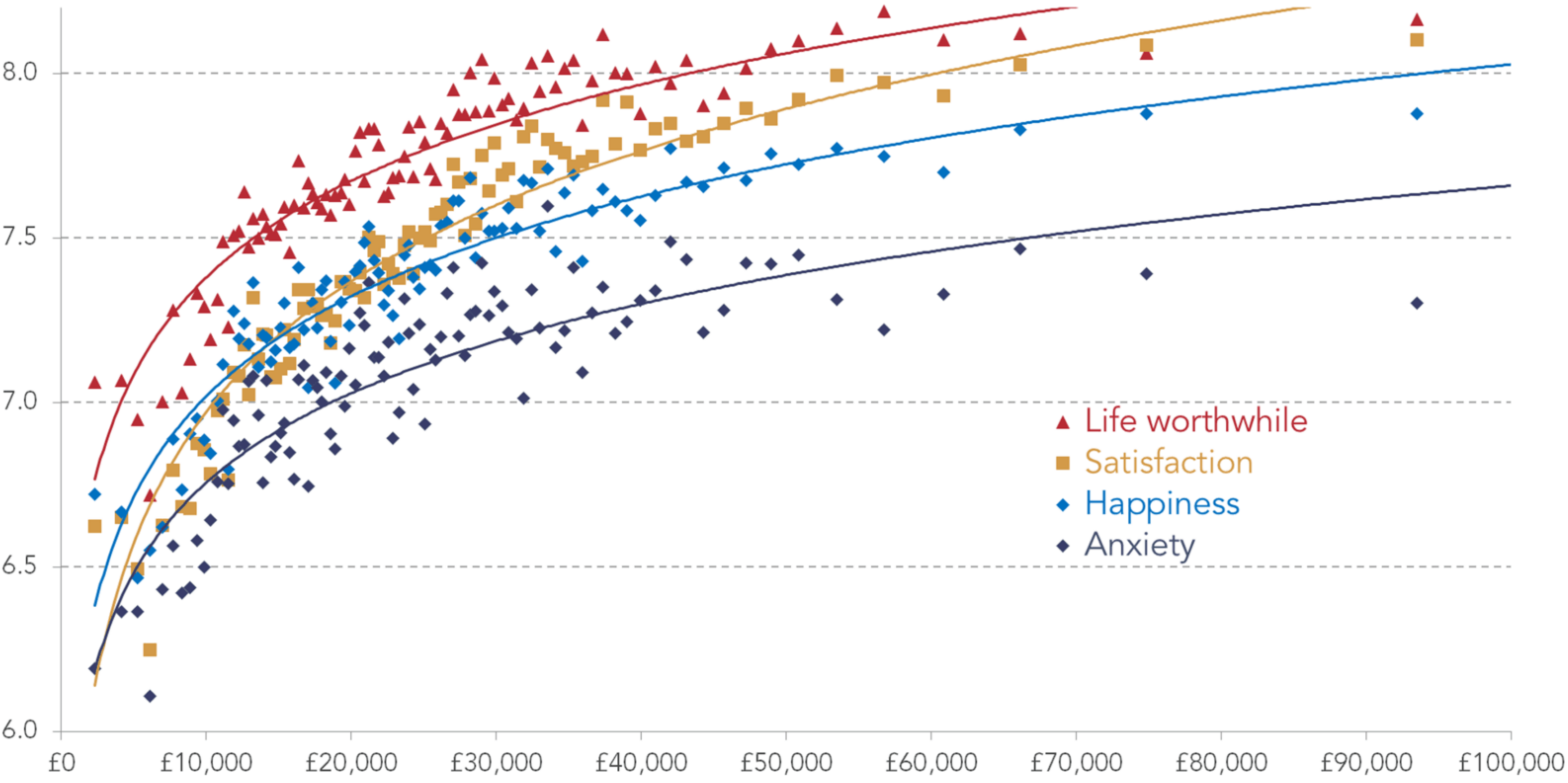
  - Intuition: does money buy you happiness? Well, *depends*.

Average subjective happiness by equivalised household income percentile (after housing costs): UK, 2014-16

RF



Notes: Each dot represents the average level of well-being for a percentile of household income (measured after housing costs), ranging from percentile 1 on the far left of the chart to percentile 100 on the far right. The lines are logarithmic lines of best fit.
Source: RF analysis of DWP, *Family Resources Survey*; pooled data for 2014-15 to 2016-17

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- **Non-linearities**
  - Intuition: does money buy you happiness? Well, *depends*.
  - Modelling strategy: **polynomial terms** + **log transformations**

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- Standard errors represent the **uncertainty** of the coefficient estimate. P-value summarise our evidence against the null that the coefficient is zero in the population.

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* When a variable is nominal, each category will have its own coefficient, which refers to the **expected difference** in the outcome between that category and the 'reference group'.
* Standard errors represent the **uncertainty** of the coefficient estimate. P-value summarise our evidence against the null that the coefficient is zero in the population.
* Unbiased estimation and inference are only valid under some heroic assumptions. Most significantly: **exogeneity**.

Interactions

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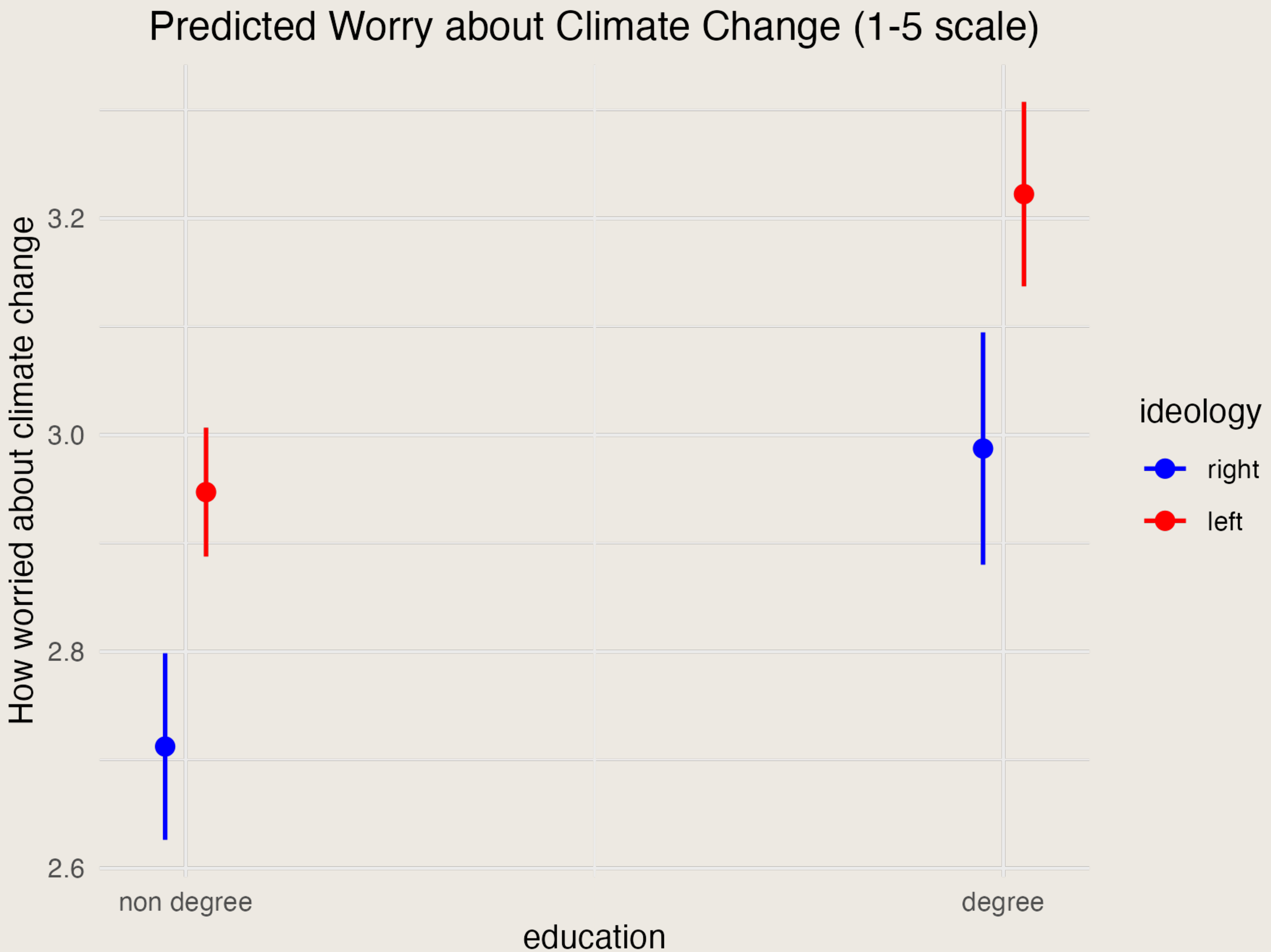
  - Ideology may be partly endogenous to education, but for now let's make peace with that, and fit:
  - Climate Worry $= \alpha + \beta_1$ Degree $+ \beta_2$ Left $+ \epsilon$

Example: Regression Table

	Dependent variable:
	wrclmch
educationdegree	0.275***
	(0.049)
ideologyleft	0.235***
	(0.049)
Constant	2.712***
	(0.044)
Observations	1,699
R2	0.031
Adjusted R2	0.030
Residual Std. Error	0.923 (df = 1696)
F Statistic	27.511*** (df = 2; 1696)
Note:	*p<0.1; **p<0.05; ***p<0.01

Example: Predicted Values Plot



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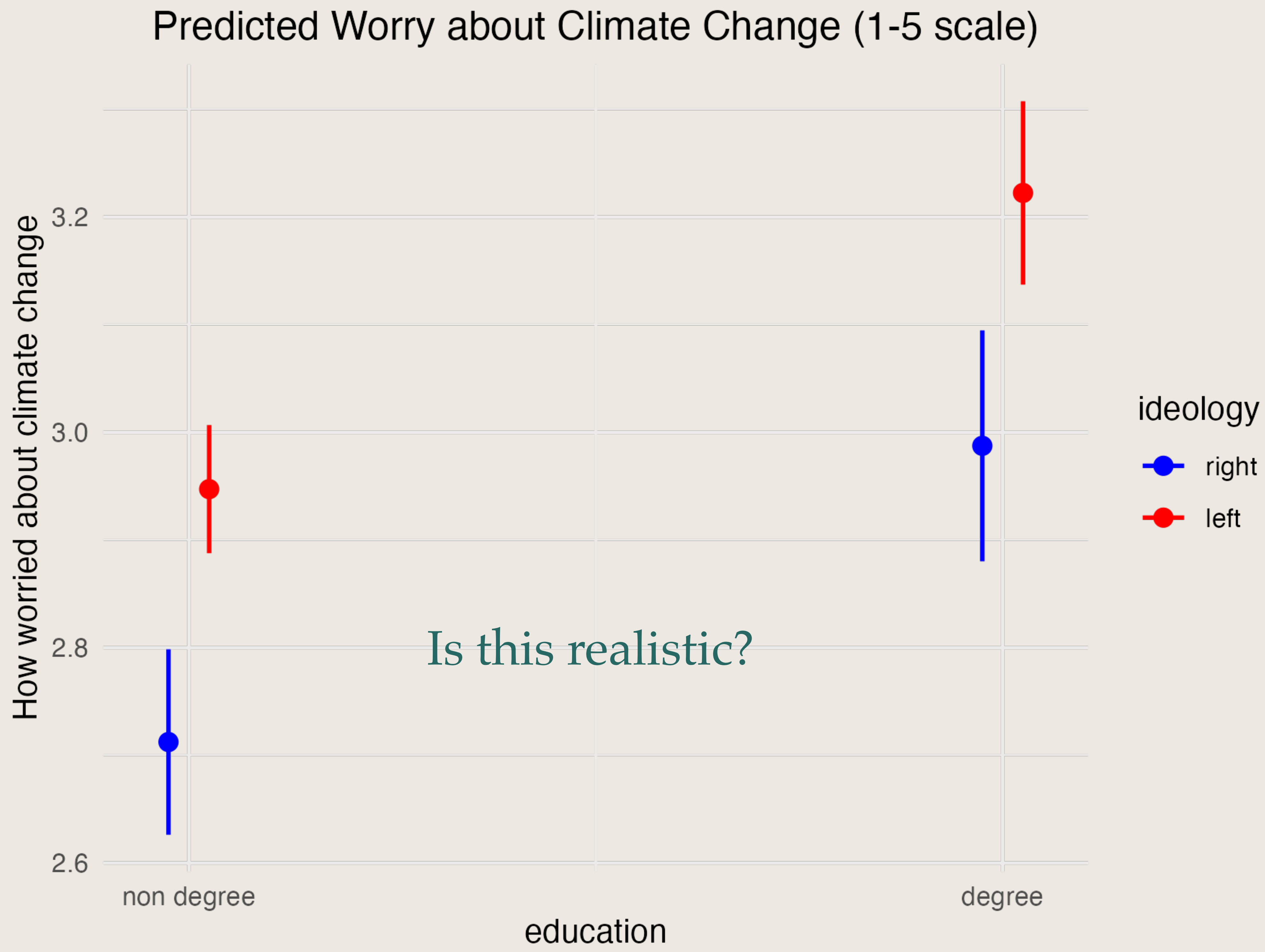

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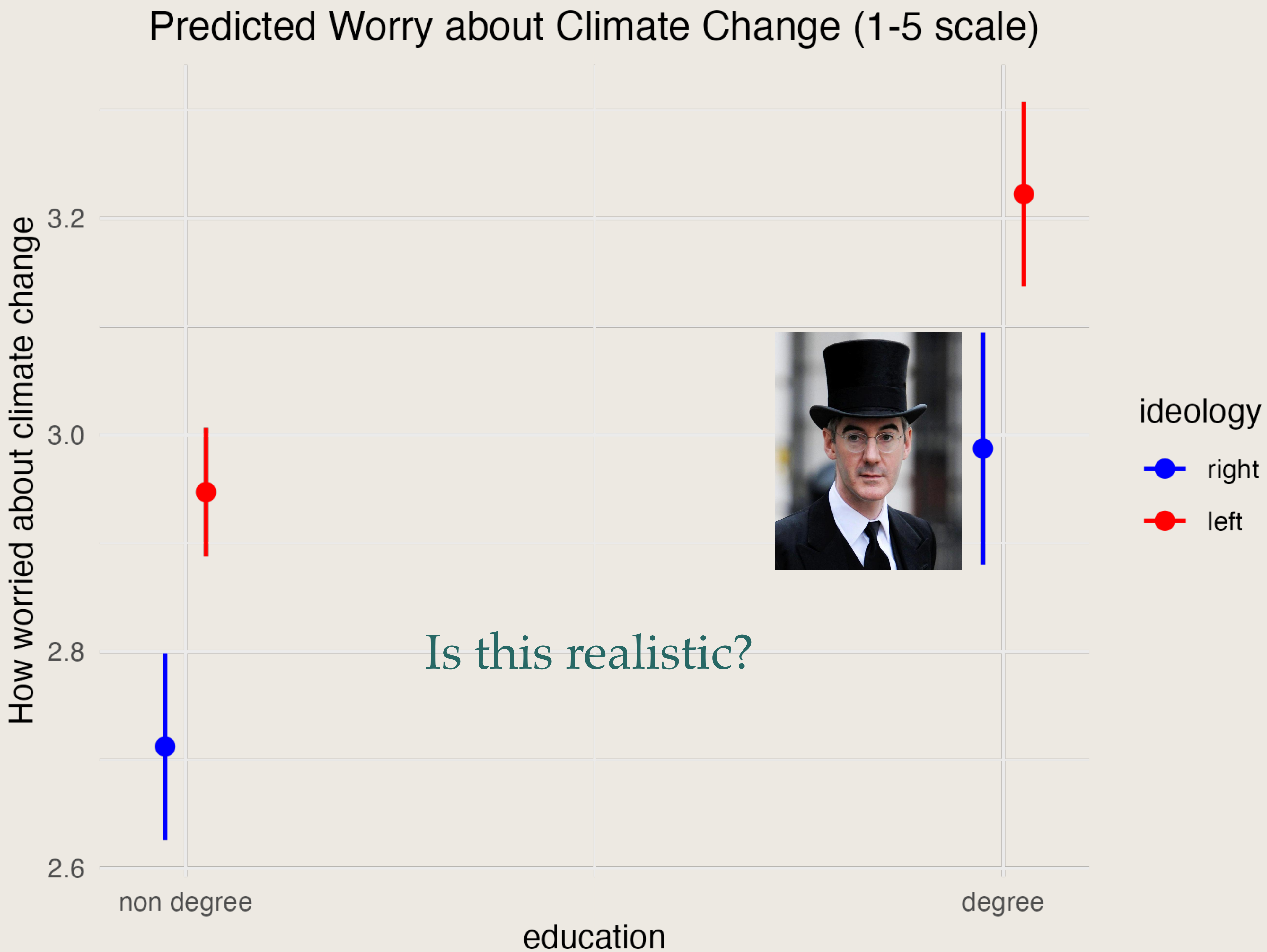

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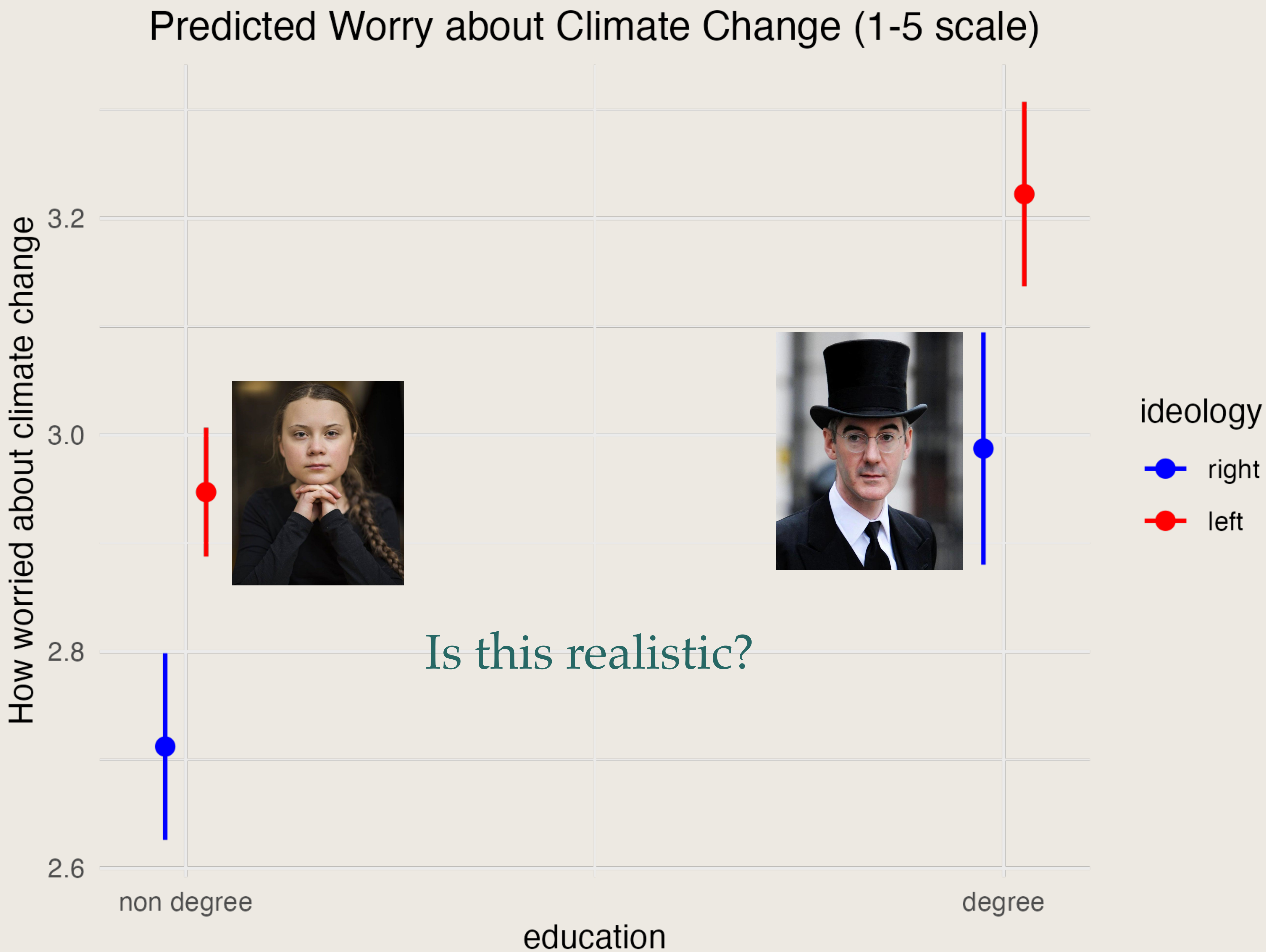

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	Dependent variable:
	Climate Worry (1–5)
Intercept	2.793*** (0.05)
Degree	−0.012 (0.09)
Left	0.121** (0.06)
Degree × Left	0.398*** (0.11)

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	Degree = 0	Degree = 1
Left = 0		
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	Degree = 0	Degree = 1
Left = 0	2.793	
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- If Degree = 0 and Left = 0, then

$$\hat{Y} = \alpha + \beta_1(0) + \beta_2(0) + \beta_3(0 \times 0) = \alpha$$

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	Degree = 0	Degree = 1
Left = 0	2.793	2.781
Left = 1		

- If Degree = 1 and Left = 0, then

$$\hat{Y} = \alpha + \beta_1(1) + \beta_2(0) + \beta_3(1 \times 0) = \alpha + \beta_1$$

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	Degree = 0	Degree = 1
Left = 0	2.793	2.781
Left = 1	2.914	

- If Degree = 0 and Left = 1, then

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	Degree = 0	Degree = 1
Left = 0	2.793	2.781
Left = 1	2.914	3.312

- If Degree = 0 and Left = 0, then

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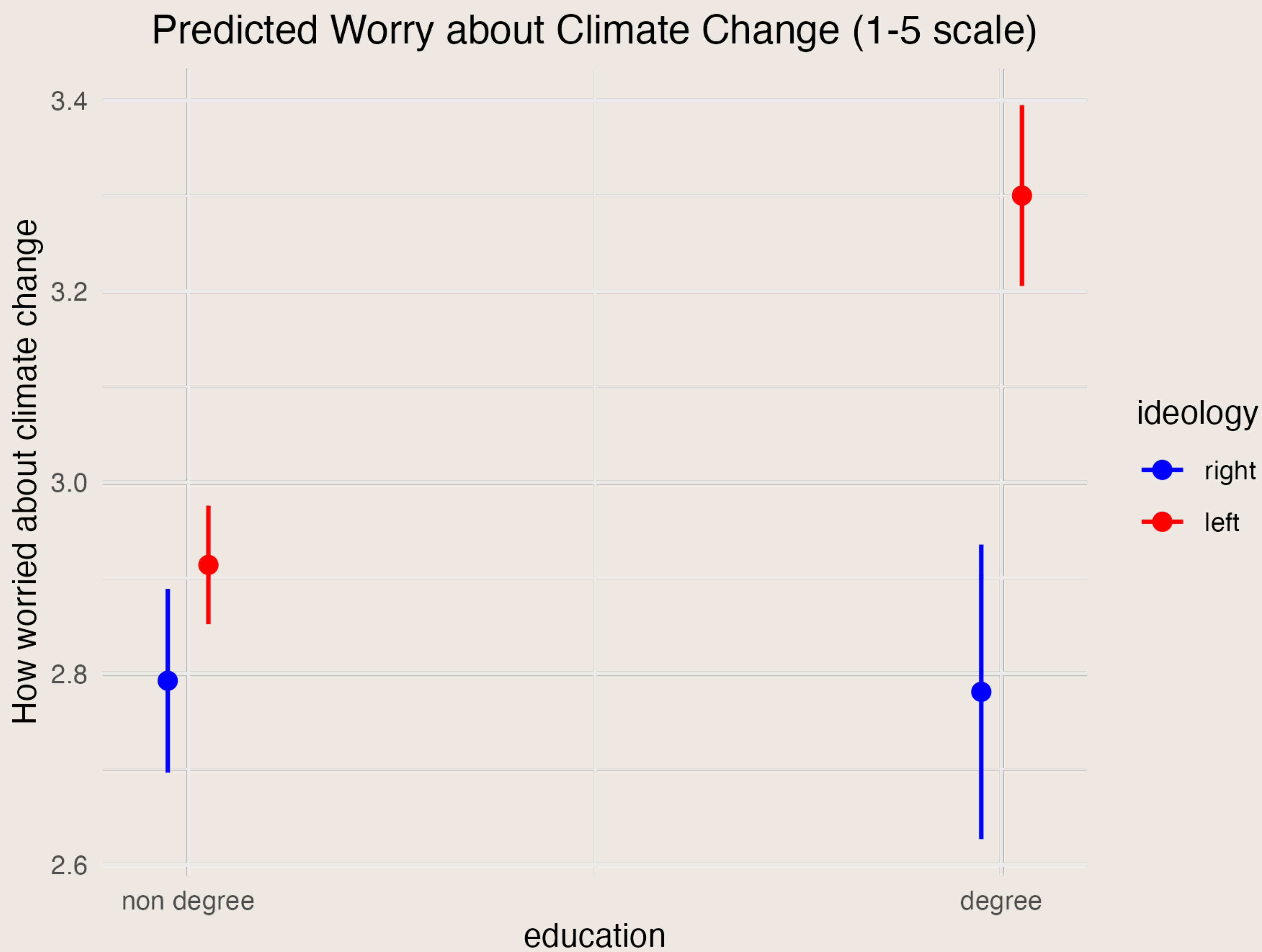

Interaction Terms in R

```
Call:
lm(formula = wrclmch ~ education + ideology + education * ideology,
    data = ess)

Residuals:
     Min       1Q   Median       3Q      Max
-2.30028 -0.79261  0.08619  0.21898  2.21898

Coefficients:
                             Estimate Std. Error t value Pr(>|t|)
(Intercept)                   2.79261    0.04900  56.997  < 2e-16 ***
educationdegree              -0.01159    0.09257  -0.125  0.90036
ideologyleft                  0.12120    0.05829   2.079  0.03776 *
educationdegree:ideologyleft  0.39805    0.10906   3.650  0.00027 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.9192 on 1695 degrees of freedom
  (260 observations deleted due to missingness)
Multiple R-squared:  0.03898,   Adjusted R-squared:  0.03727
F-statistic: 22.91 on 3 and 1695 DF,  p-value: 1.533e-14
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- This is **a really good feature** of `lm()`. Whenever you have interaction terms, you **always** want to control for the parent terms (*education* and *ideology*) as well as the interaction term.

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- There is a way of telling R to include only the interaction term (*education* × *ideology*), but it's best you don't know because this is **wrong** 99% of the times.

Interpreting Interaction Terms

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- Here: large and significant — we do have an important interaction.

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- In R, just pass the categorical variable:

```
lm(wrclmch ~ education + ideo_group + education*ideo_group, data = ess)

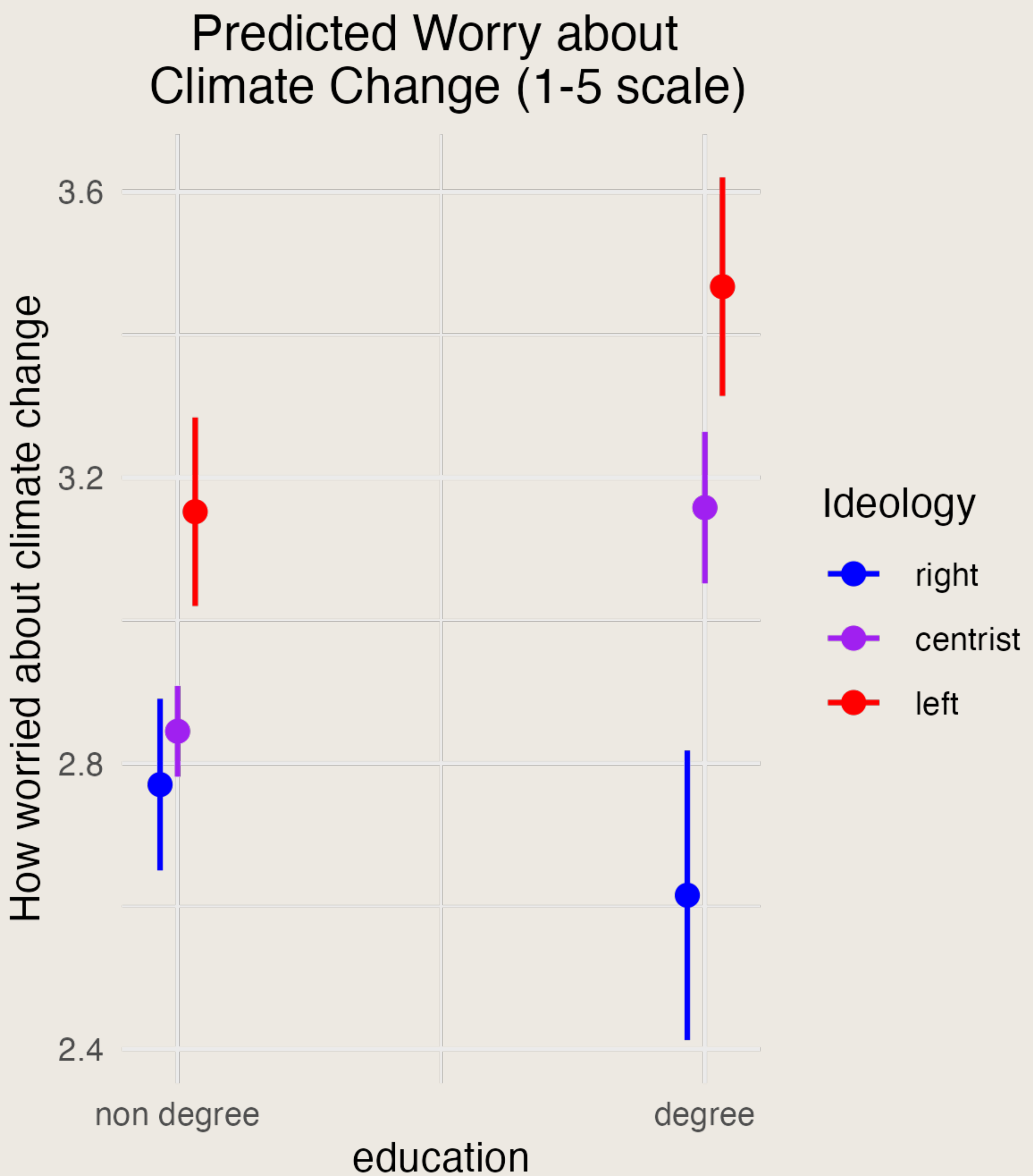
# or equivalently

lm(wrclmch ~ education*ideo_group, data = ess)
```

Categorical Moderators with More Levels

	Dependent variable:
	Climate Worry (1–5)
Intercept	2.770*** (0.061)
Degree	−0.155 (0.120)
Centrist	0.075 (0.069)
Left	0.382***(0.091)
Degree × Centrist	0.468*** (0.136)
Degree × Left	0.470***(0.148)
Observations	1,699
Adjusted R^2	0.052

Note: *p<0.1; **p<0.05; ***p<0.01



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* β_1 is the estimate for the effect of 'Degree' on 'Worry' **when 'R-L Scale' is zero** (i.e. for the most right-wing).

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- β_2 is the predicted change in 'Worry' associated with of a **one-unit increase** in 'R-L Scale' when 'Degree' is zero (i.e. for non-graduates).

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- β_1 is the estimate for the effect of 'Degree' on 'Worry' **when 'R-L Scale' is zero** (i.e. for the most right-wing).
- β_2 is the predicted change in 'Worry' associated with of a **one-unit increase** in 'R-L Scale' when 'Degree' is zero (i.e. for non-graduates).
- β_3 is tricky: it's the change in the effect of 'Degree' on 'Worry' **as we increase the value of 'L-R Scale' by one unit**. Easier to interpret significance and direction, use plots to show effect size.

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$$\text{Worry} = \alpha + \beta_1 \text{ Degree} + \beta_2 \text{R-L Scale} + \beta_3 \text{ (R-L Scale} \times \text{Degree)} + \epsilon$$

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	Dependent variable:
	Climate Worry (1–5)
Intercept	
Degree	
R-L Scale	
Degree × R-L Scale	

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Dependent variable:

	Climate Worry (1–5)
Intercept	2.544*** (0.075)
Degree	
R-L Scale	
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$$\text{Worry} = \alpha + \beta_1 \text{ Degree} + \beta_2 \text{R-L Scale} + \beta_3 \text{ (R-L Scale} \times \text{Degree)} + \epsilon$$

β_1 = effect of 'Degree' on 'Worry' when 'R-L Scale' is zero

	Dependent variable:
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Degree	−0.116 (0.142)
R-L Scale	
Degree × R-L Scale	

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β_1 = effect of 'Degree' on 'Worry' when 'R-L Scale' is zero

β_2 = effect of a one-unit increase in 'R-L Scale' on 'Worry' when 'Degree' is zero

	Dependent variable:
	Climate Worry (1–5)
Intercept	2.544*** (0.075)
Degree	−0.116 (0.142)
R-L Scale	0.068***(0.014)
Degree × R-L Scale	

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β_1 = effect of ‘Degree’ on ‘Worry’ when ‘R-L Scale’ is zero

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β_3 = change in the effect of ‘Degree’ on ‘Worry’ as we increase the value of ‘L-R Scale’ by one unit.

	Dependent variable:
	Climate Worry (1–5)
Intercept	2.544*** (0.075)
Degree	−0.116 (0.142)
R-L Scale	0.068***(0.014)
Degree × R-L Scale	0.068***(0.025)

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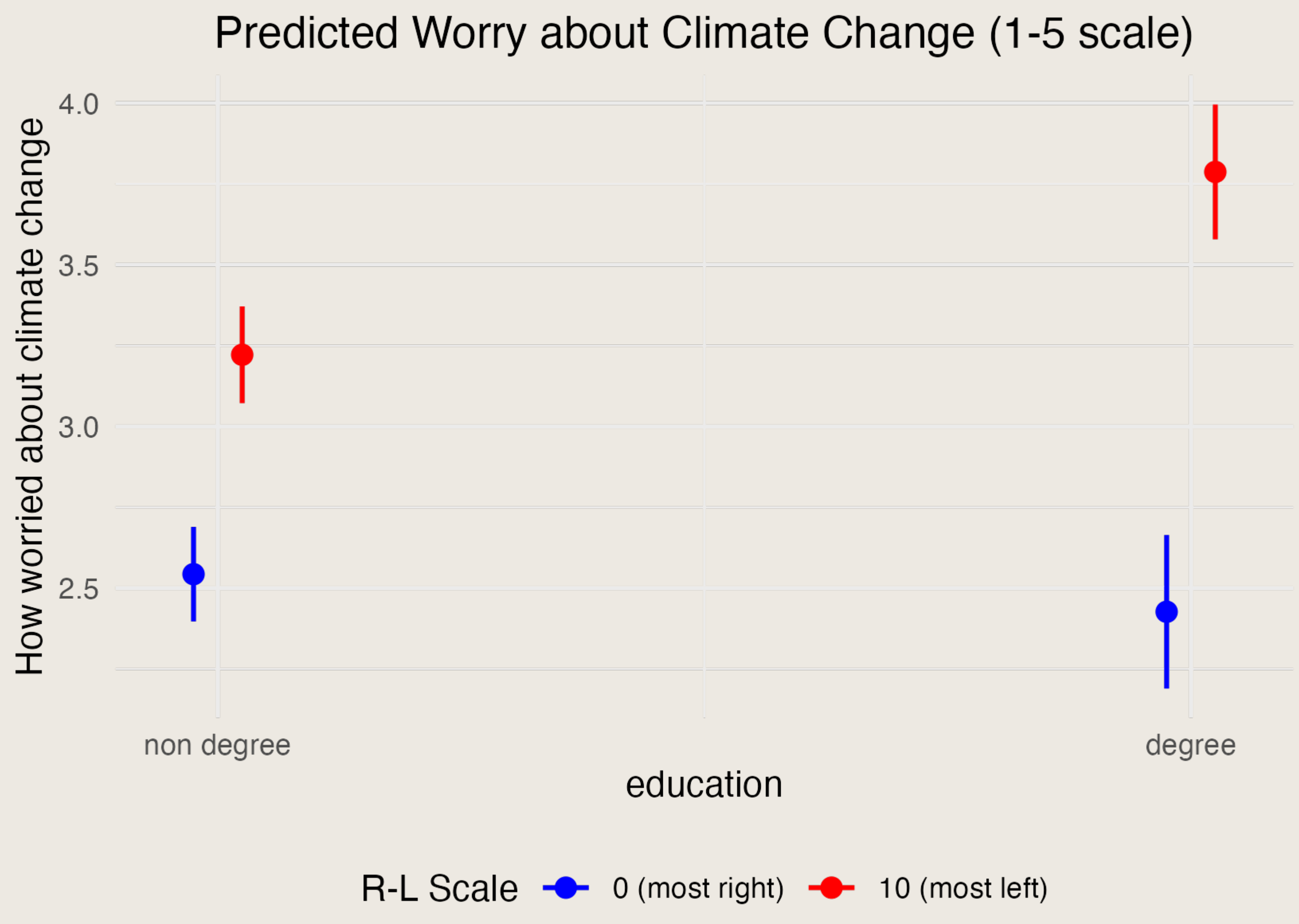

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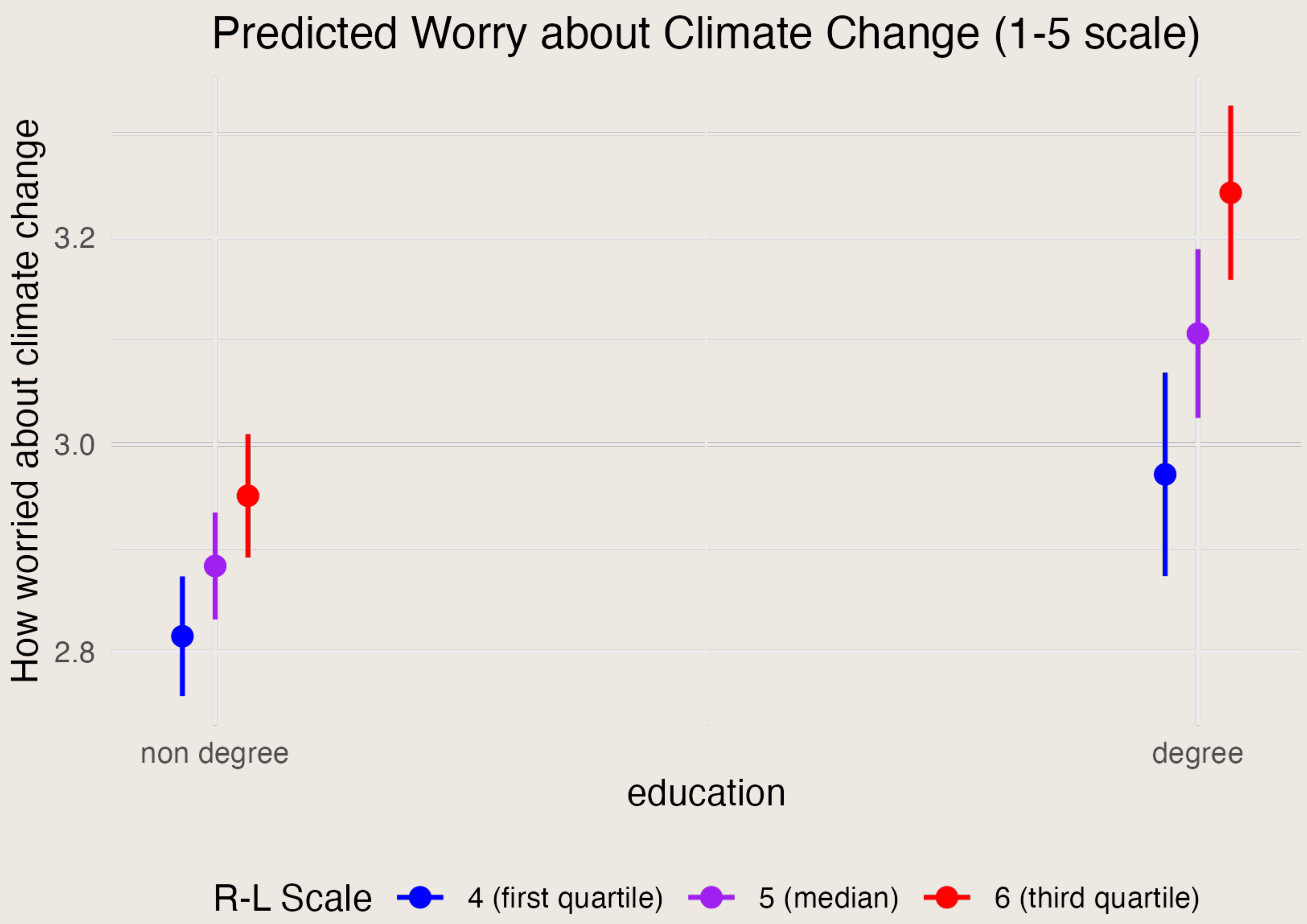

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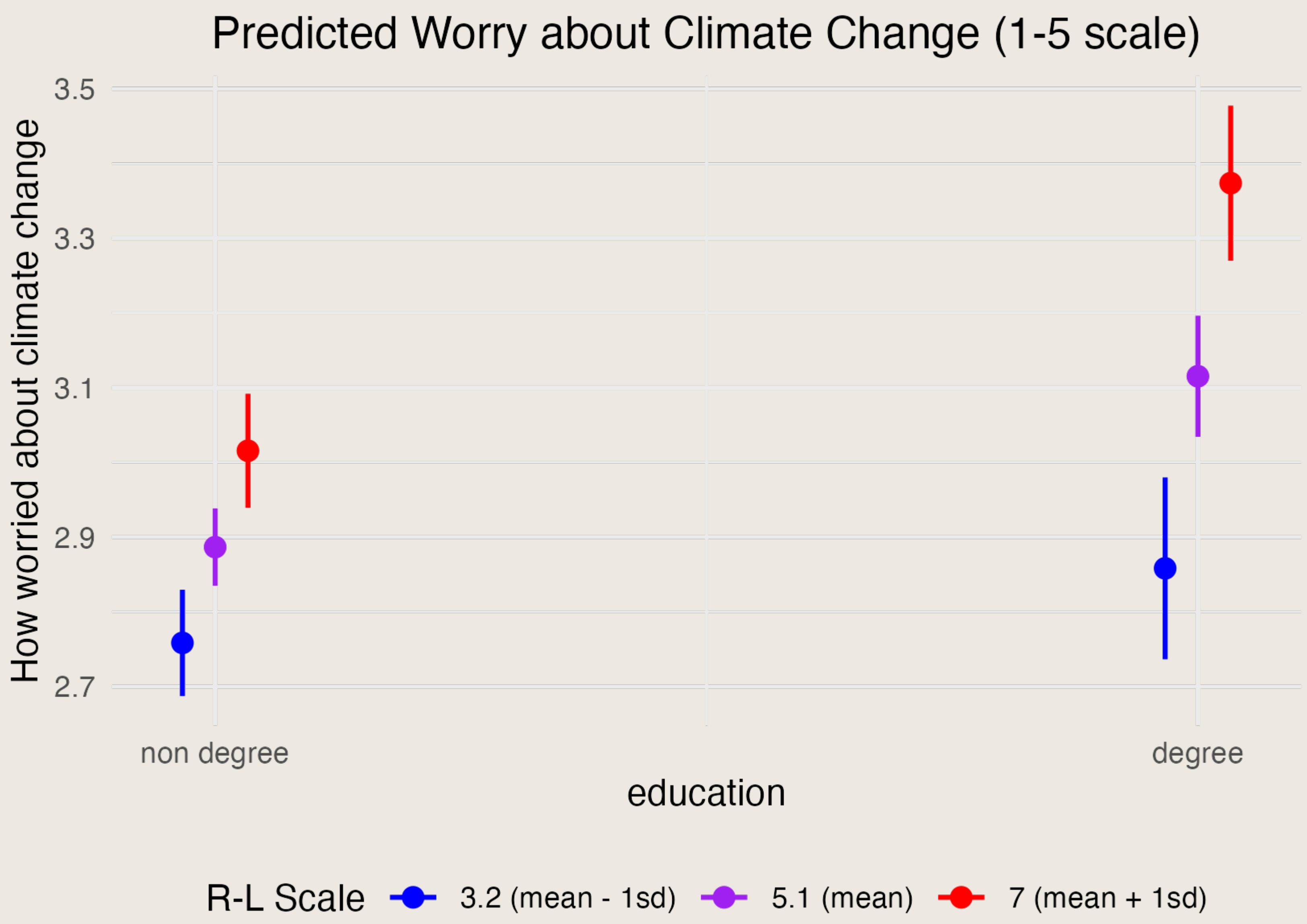

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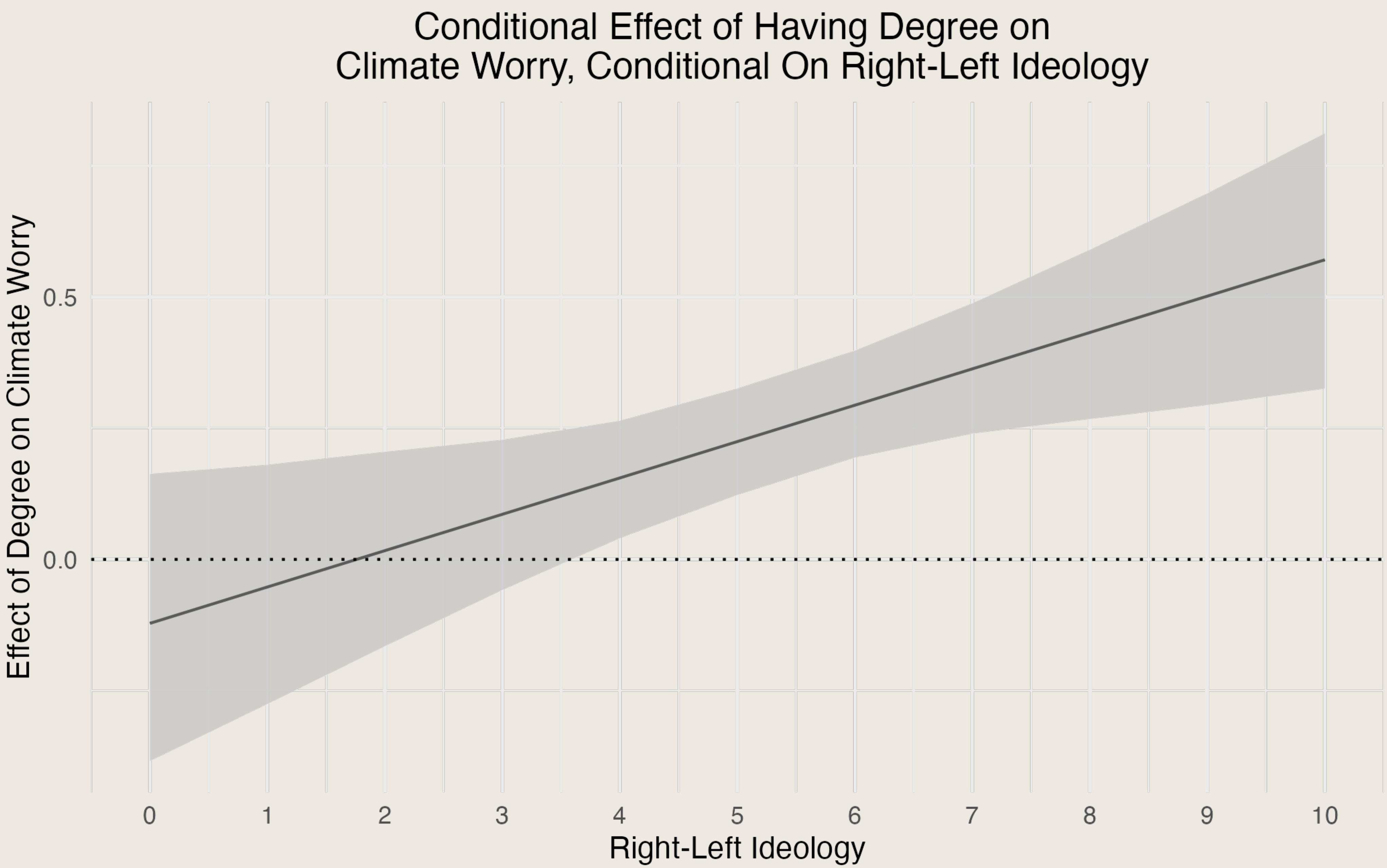

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- The interaction term's coefficient is the estimated **change in the effect of one year of education** on Climate Worry, associated with a **one-point increase in the R-L scale**.

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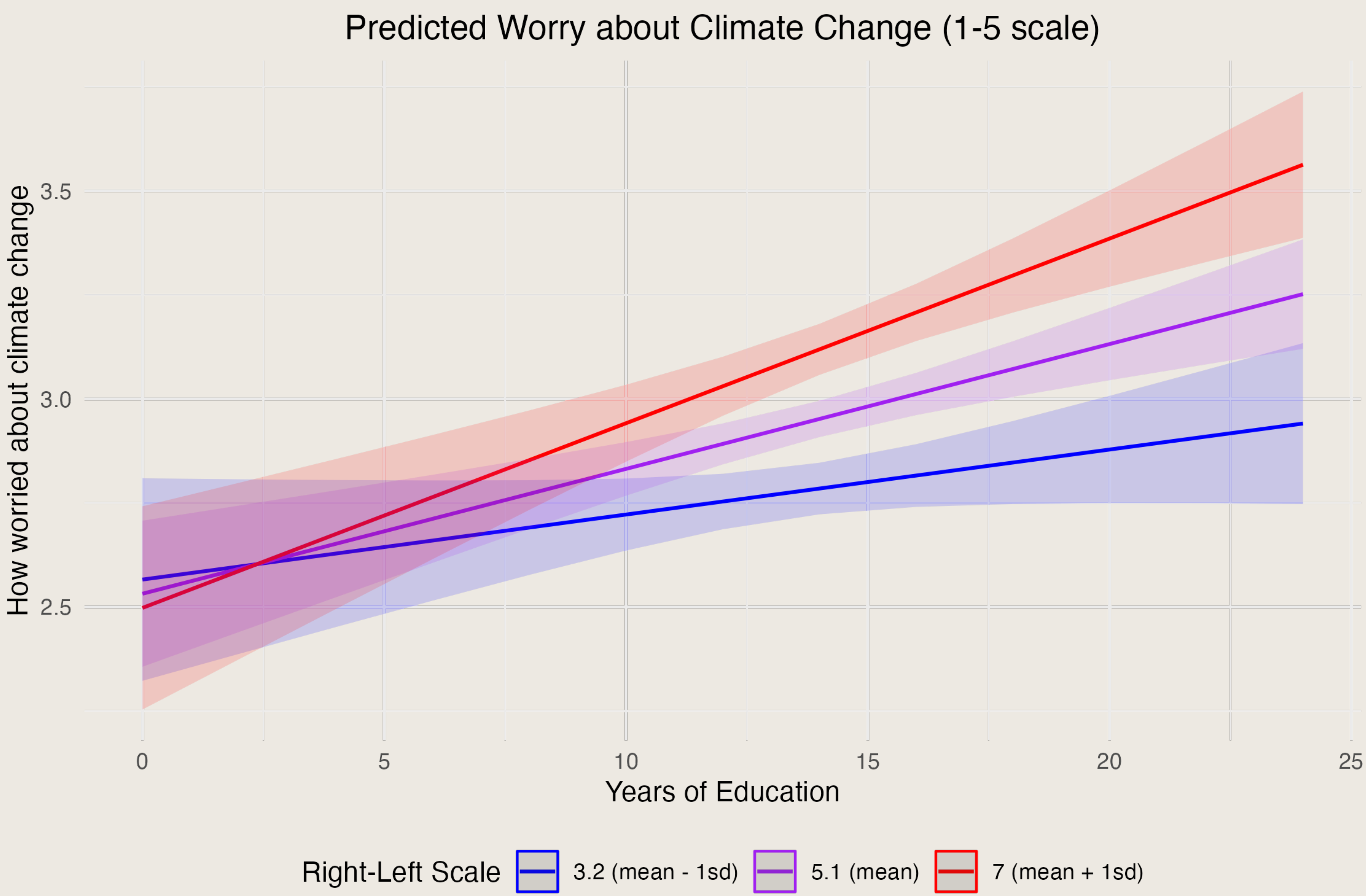
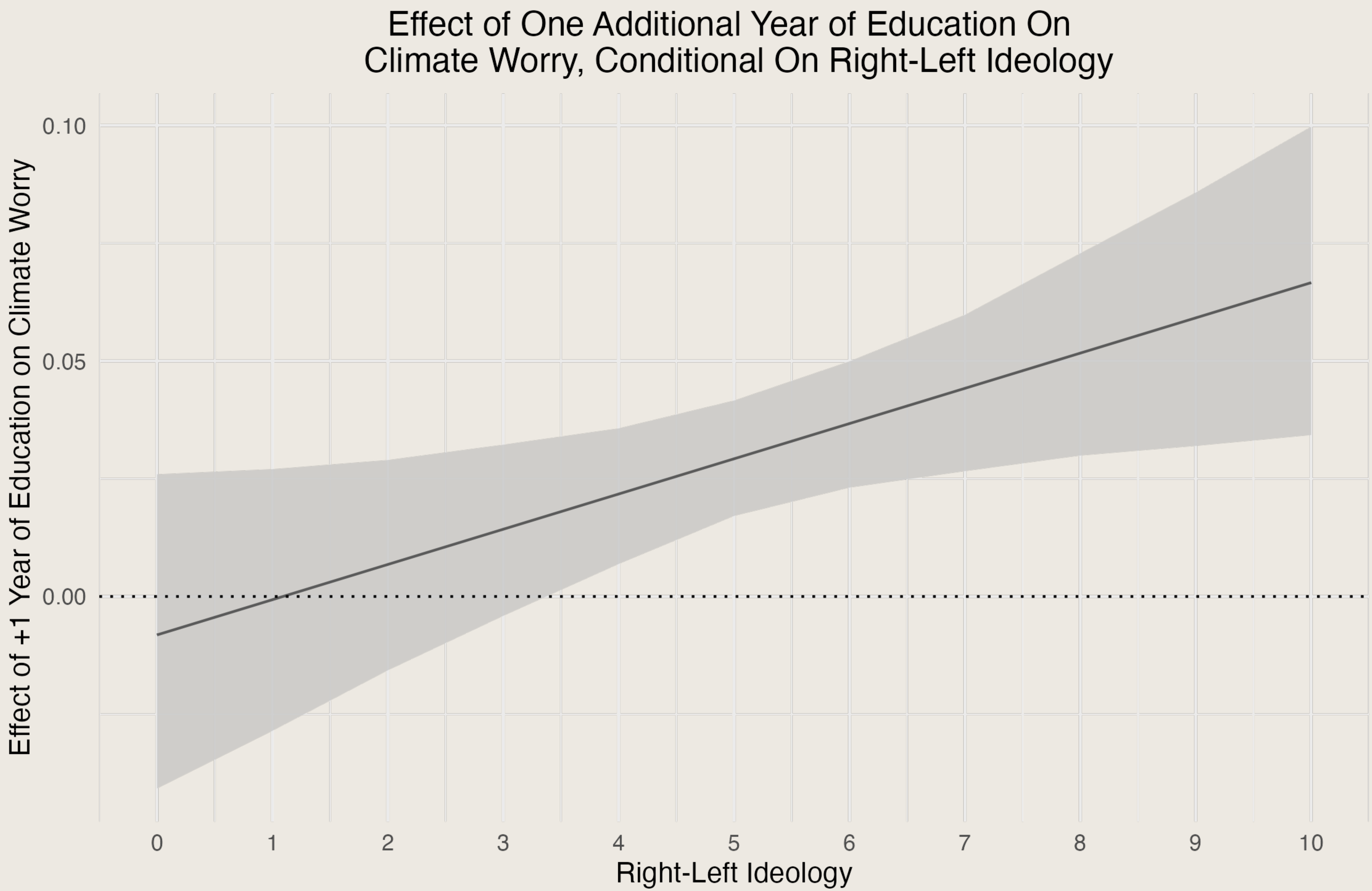
Predicted Values Plot



Conditional Effects Plot (aka Marginal Effect Plot)



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- Potentially **infinite** combinations of interaction terms. You will get ‘lucky’ and find something significant at some point.

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* Our main effects are already noisy, because they're estimated with uncertainty.
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* You need very large sample sizes to estimate an interaction effect precisely (16× larger than for a main effect).

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Check if you understand (1)

- Does 'winning' (i.e. voting for the party that forms the government) make people feel happier?

	Random Intercept, Interaction
Winner	.101*** (.021)
Corruption	–.079*** (.029)
Winner*Corruption	–.014** (.007)
Nonvoter	–.034** (.018)
Left-right self-placement	.018*** (.003)
Constant	3.166*** (.522)
Variance components	
Country	.018*** (.006)
Individual	.435*** (.005)
–2 log likelihood	26,133.8
N at Level 1	12,996
N at Level 2	16

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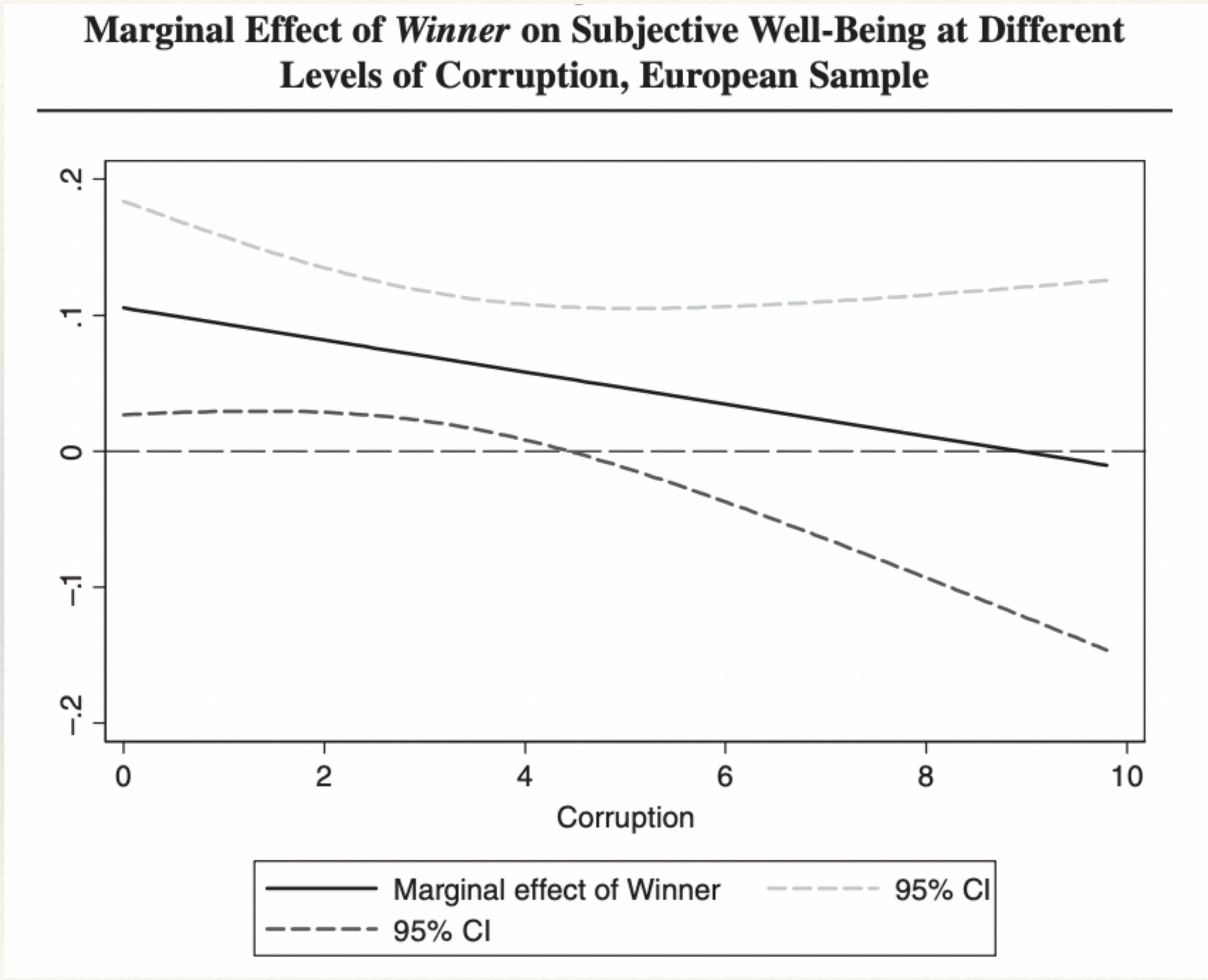

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Check if you understand (2)

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	Anger and Party Threat	
	1	2
Partisan strength	– .01 (.03)	.01 (.03)
Partisan identity	–	– .07 (.07)
Party threat/reassurance	.26 (.06)***	.03 (.08)
Partisan strength × threat/reassurance	.10 (.04)**	– .01 (.04)
Partisan identity × threat/reassurance	–	.44 (.09)***
Ideological issue intensity	.06 (.05)	.07 (.05)
Ideological intensity × threat/reassurance	– .03 (.07)	– .03 (.07)
Knowledge	– .19 (.10)*	– .19 (.09)**
Gender (male)	– .04 (.02)**	– .03 (.02)*
Education	– .05 (.04)	– .04 (.04)
Age (decades)	.01 (.01)	.00 (.01)
Constant	.42 (.11)***	.46 (.11)***
Adj. R^2	*0.22*	*0.24*
N	*1482*	*1482*

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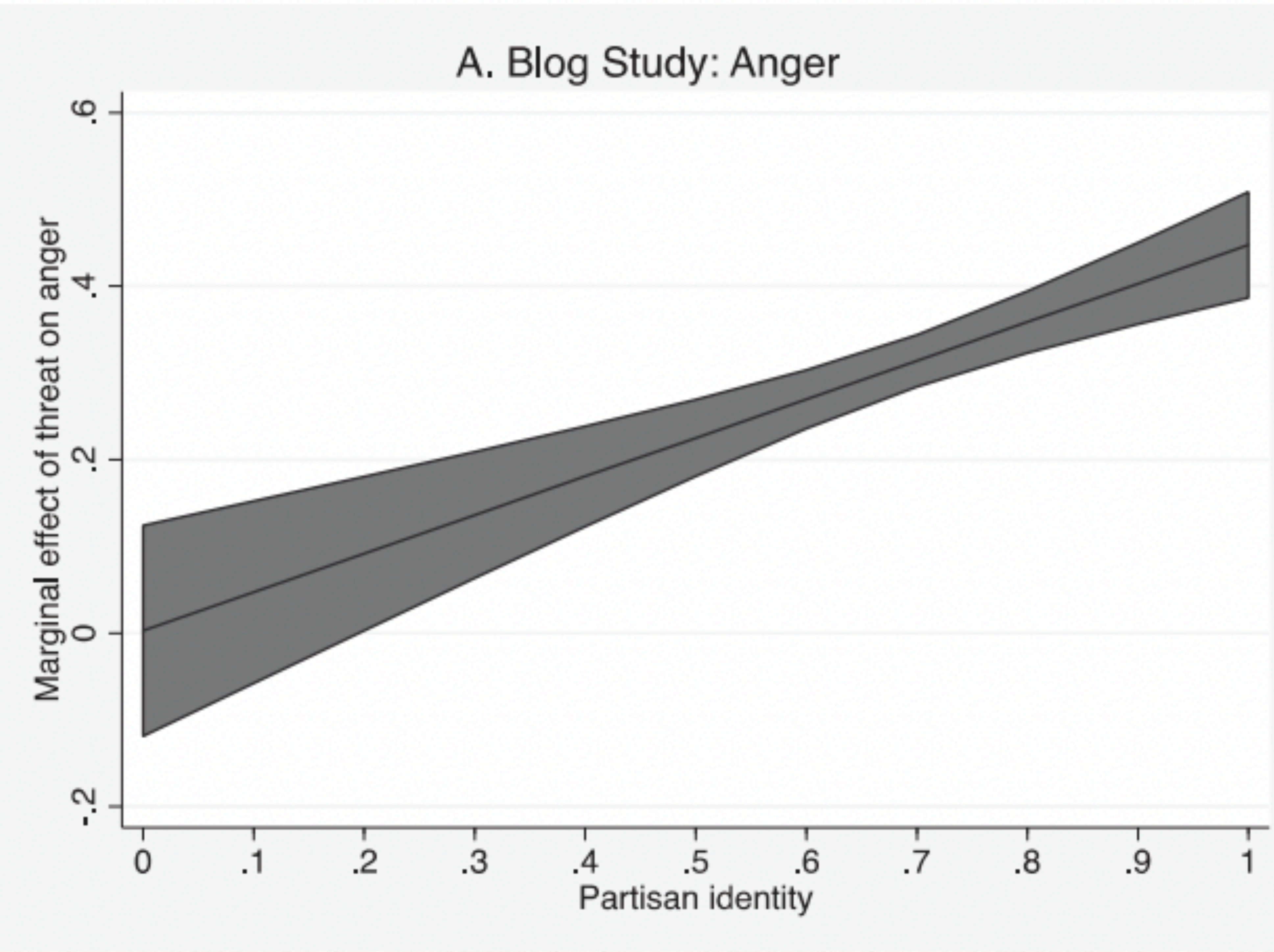

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- Both approaches are consistent with linearity assumptions: regression are still 'linear in the βs'.

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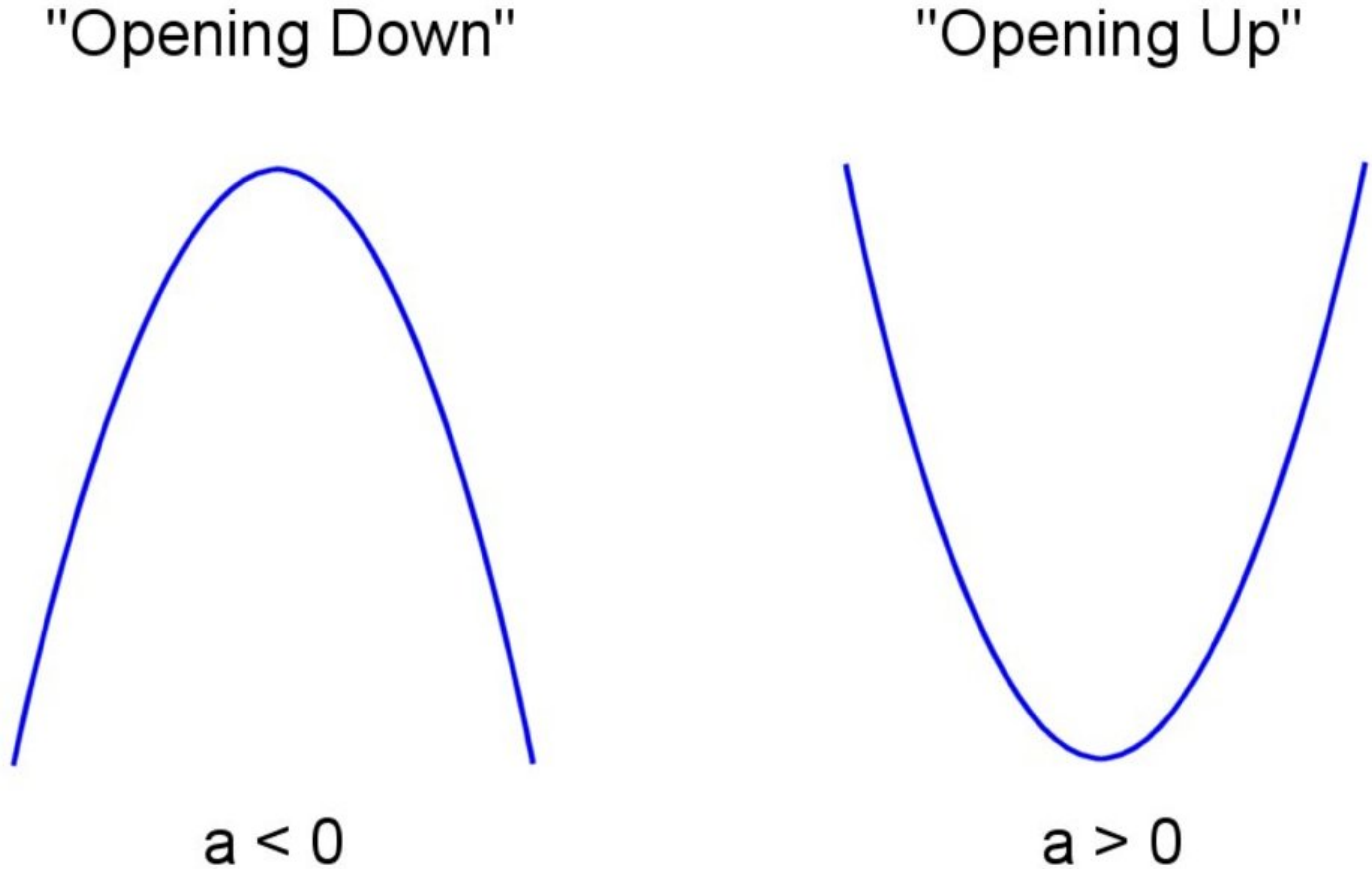
- Characteristics of a parabolic curve:
- It is **U-shaped** ('opening up') if $\beta_2 > 0$. It is **n-shaped** ('opening down') if $\beta_2 < 0$.
- It has **one** bend, known as its vertex, given by $-\dfrac{\beta_1}{2\beta_2}$



The coefficient of x^2 determines whether the parabola opens up or down

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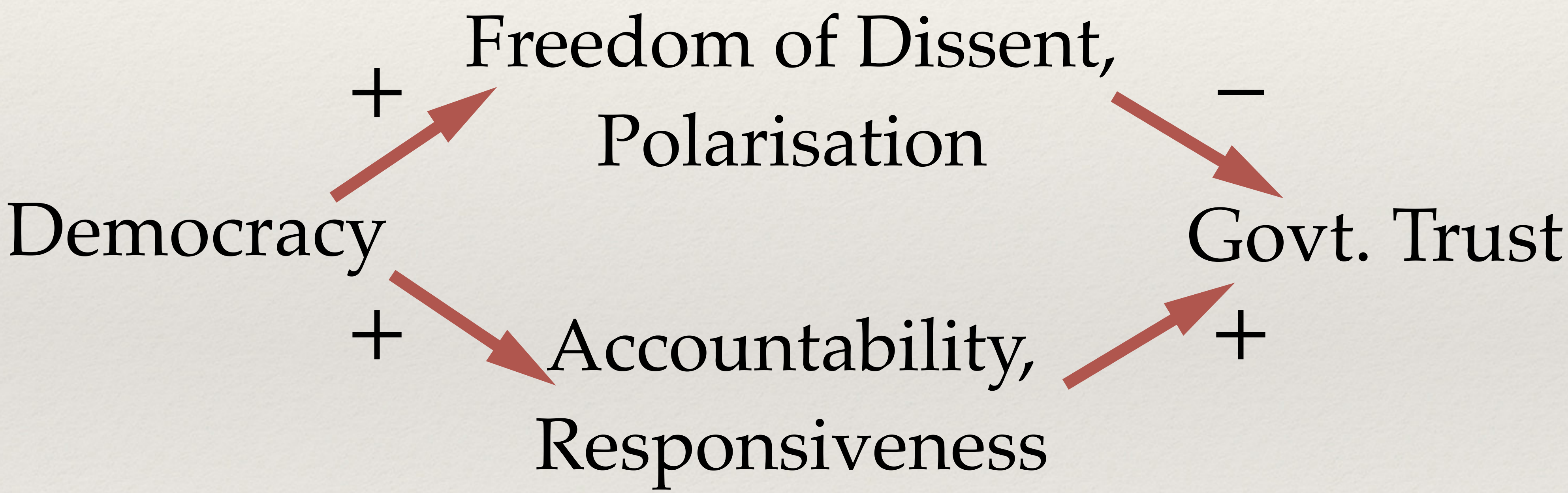

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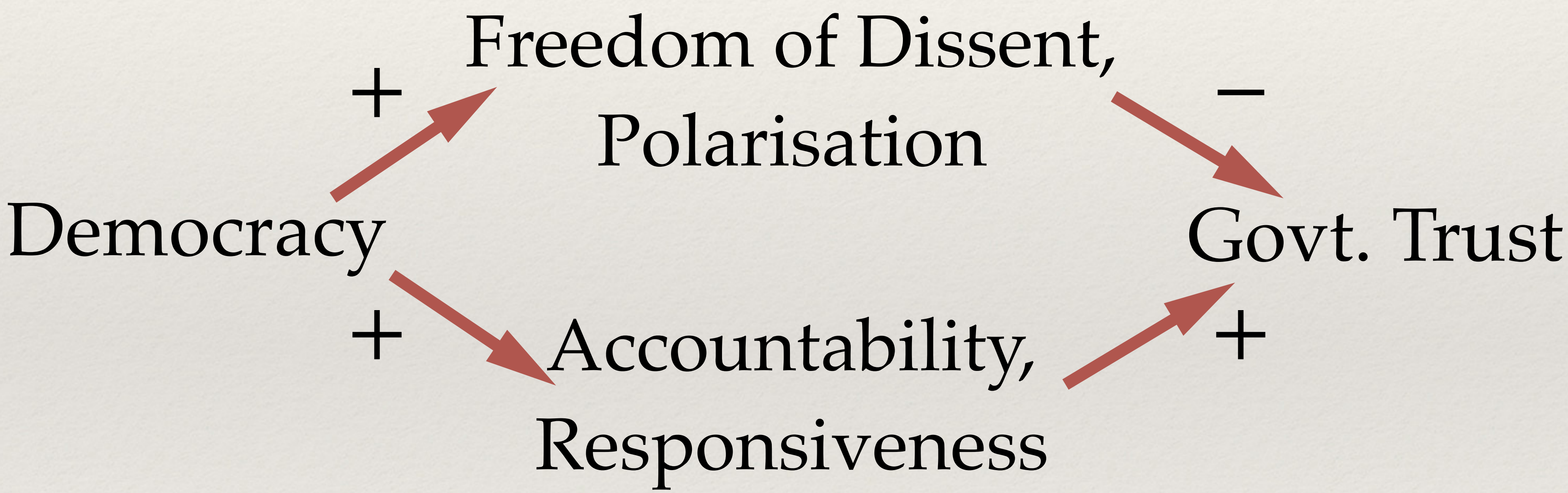

* We gather data on **Democracy** (0-10 scale) from V-Dem, and on the average country-level **Trust in Government** (1 = none at all, 4 = a great deal) from the World Values Survey (WVS).

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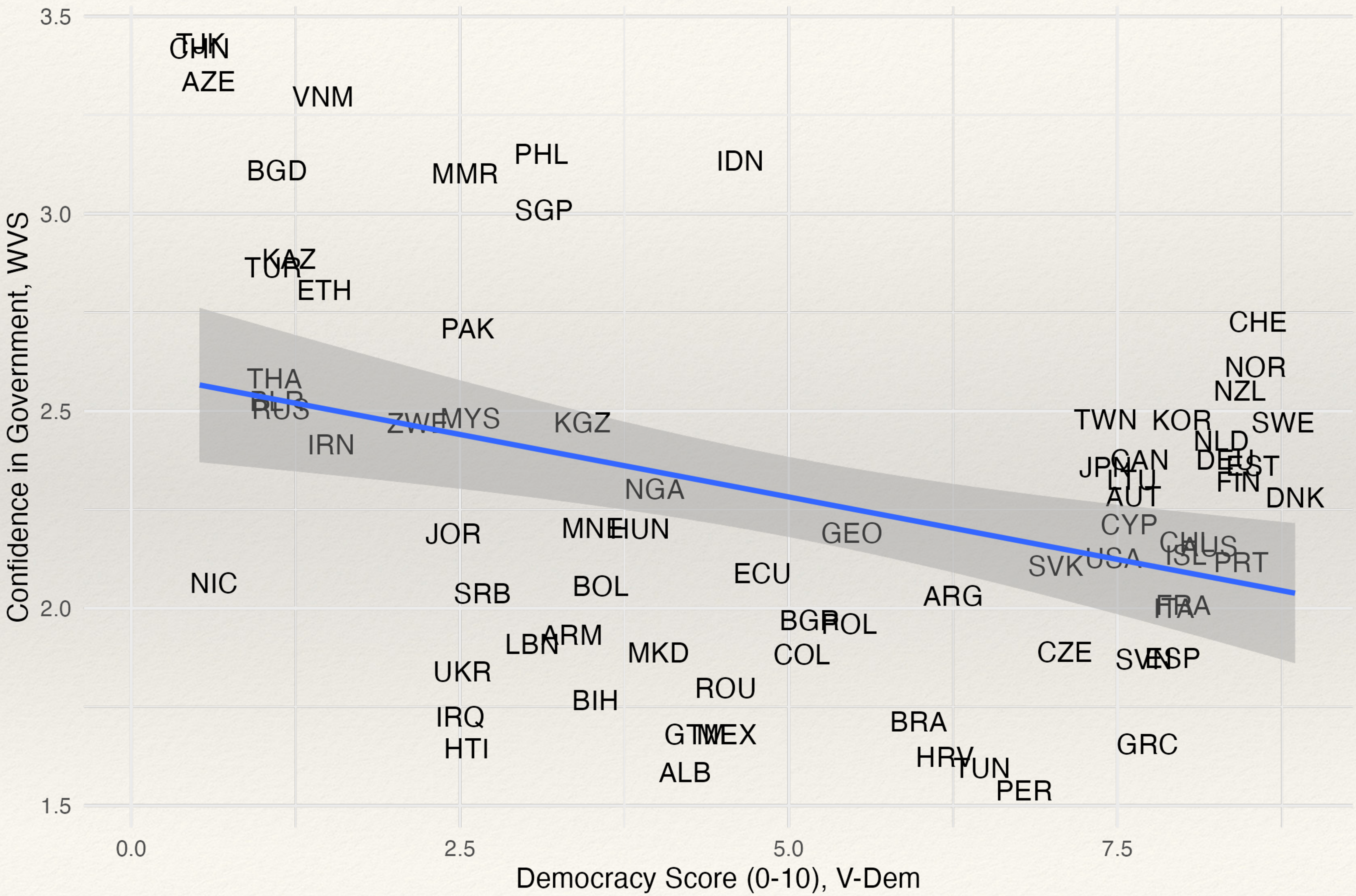

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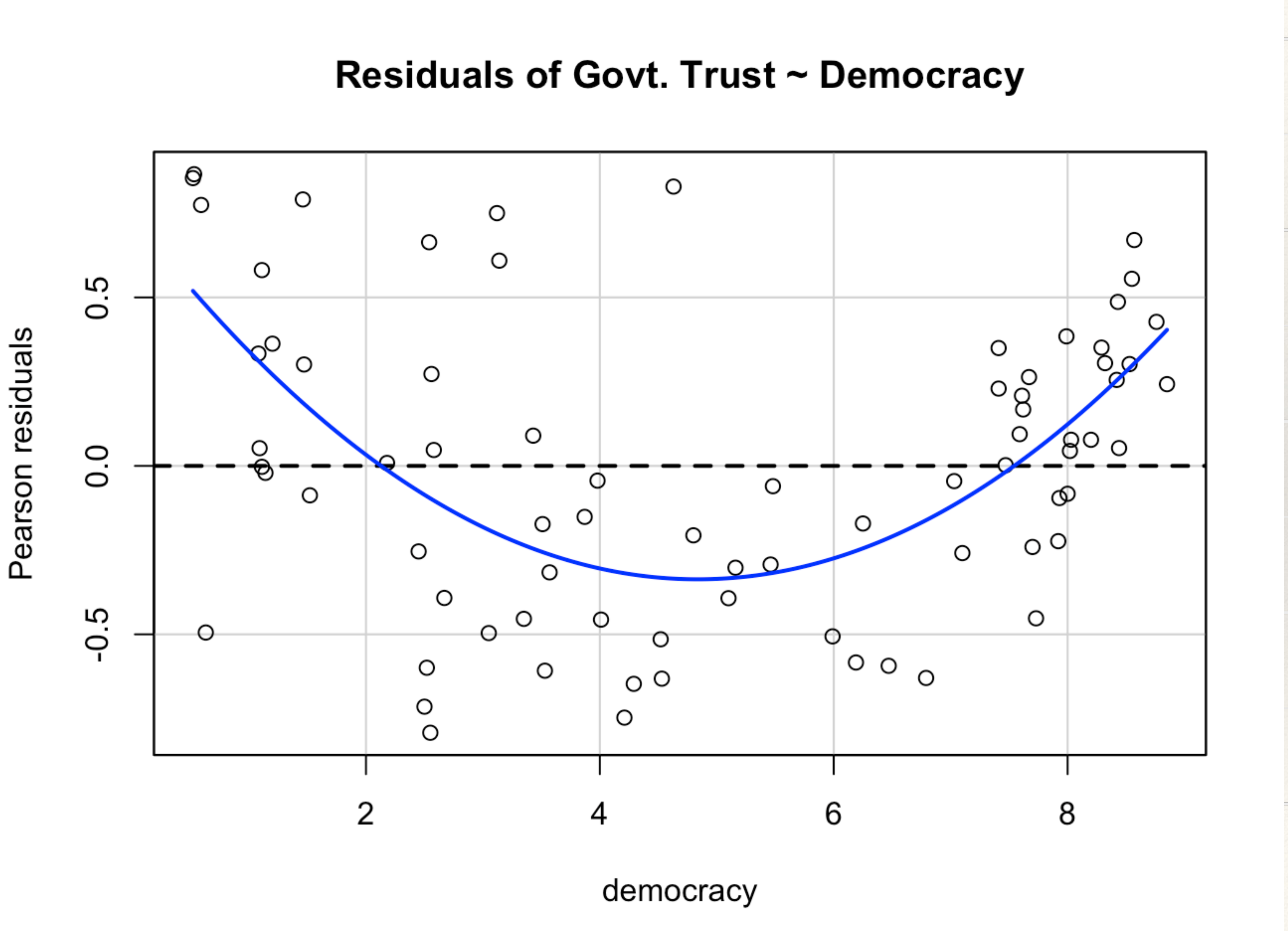

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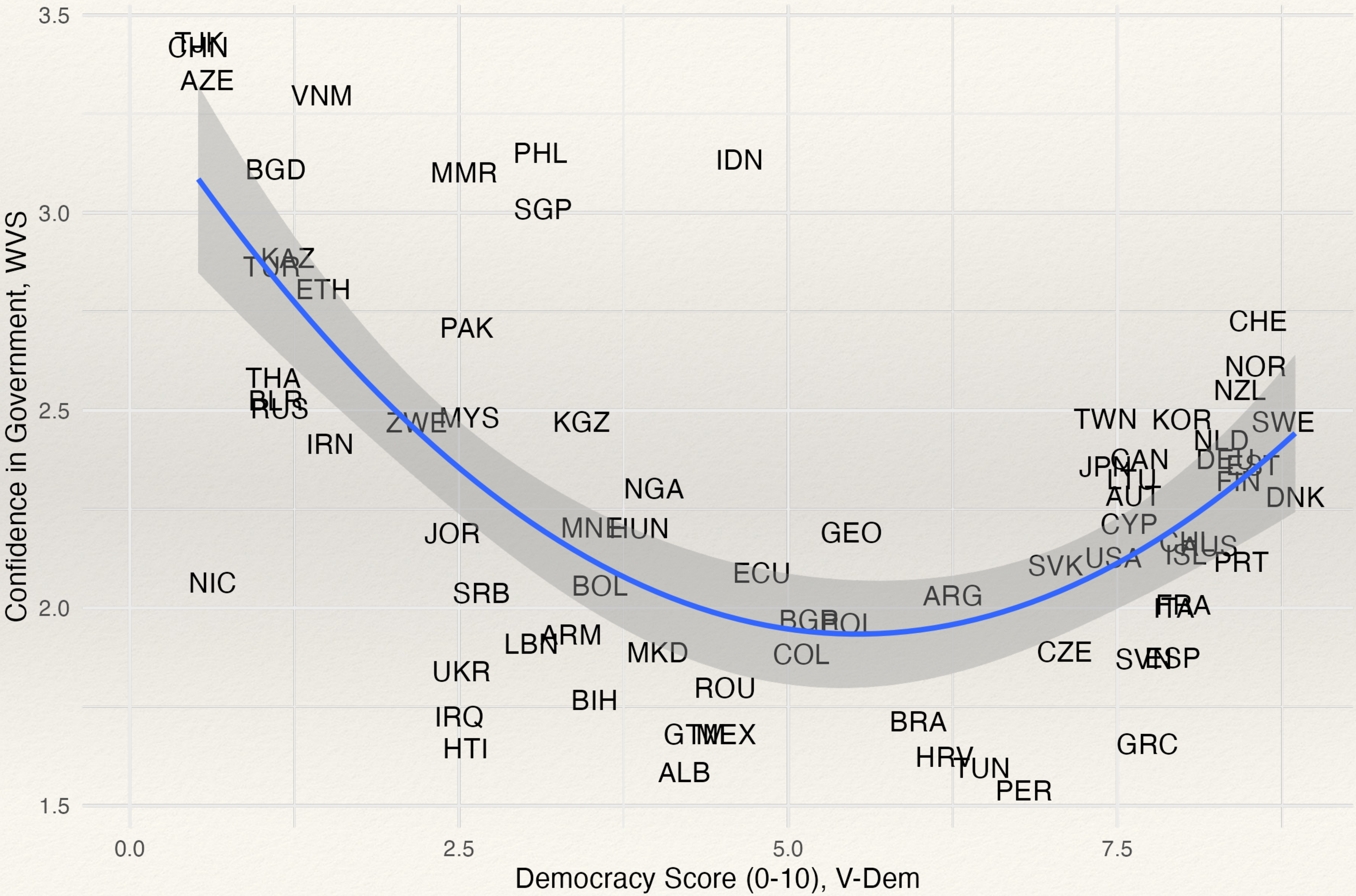

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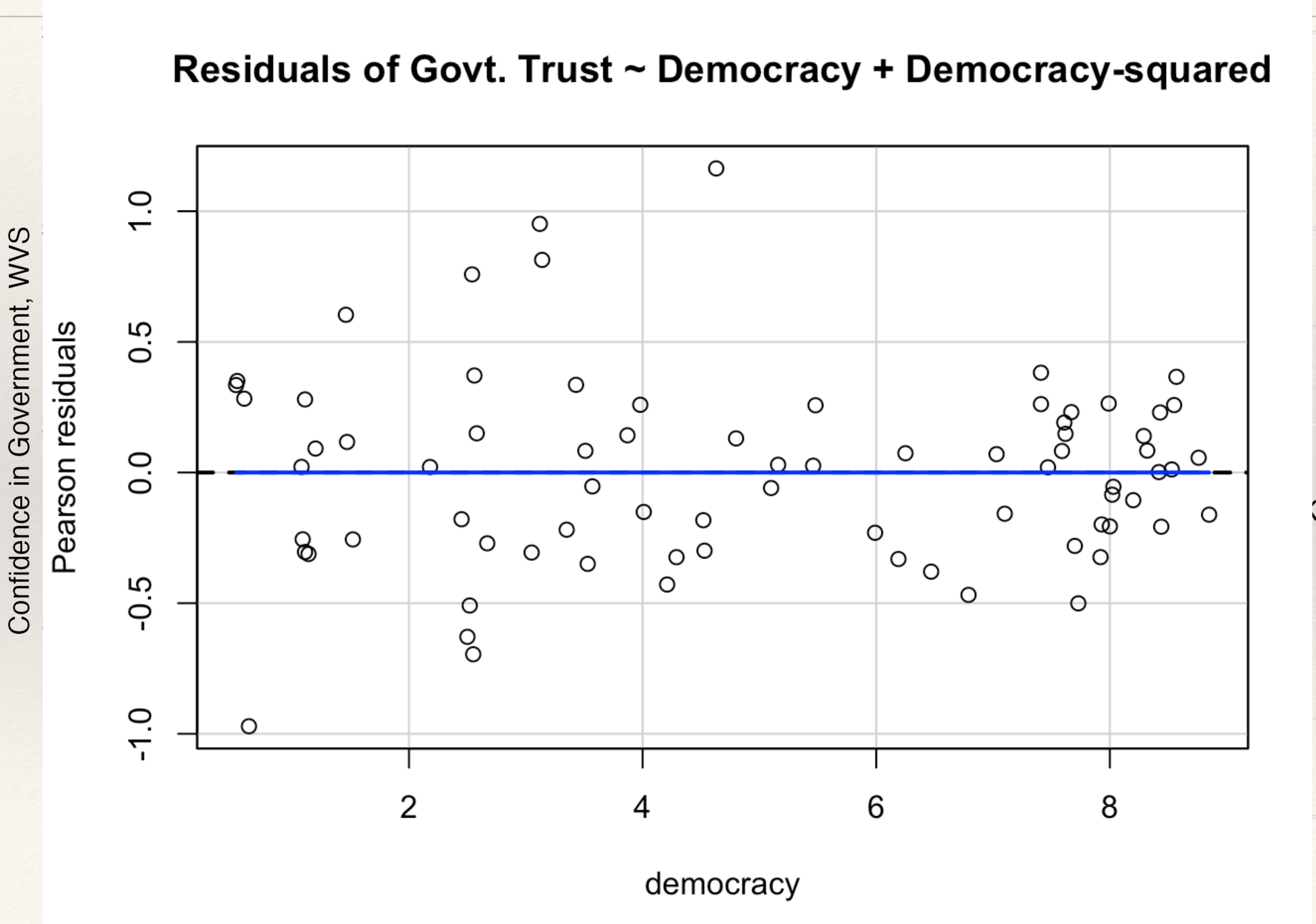
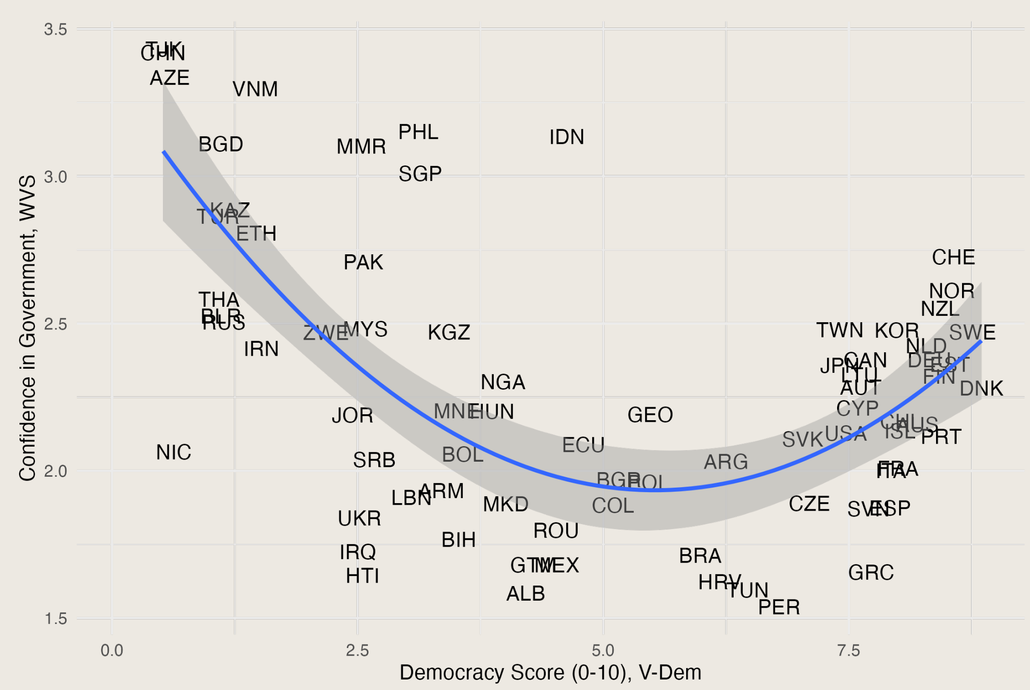


Second-Degree Polynomial: Coefficients

	Dependent variable:
	Govt. Trust (1–4)
Intercept	3.337*** (0.152)
Democracy	-0.508*** (0.076)
Democracy²	0.046*** (0.008)



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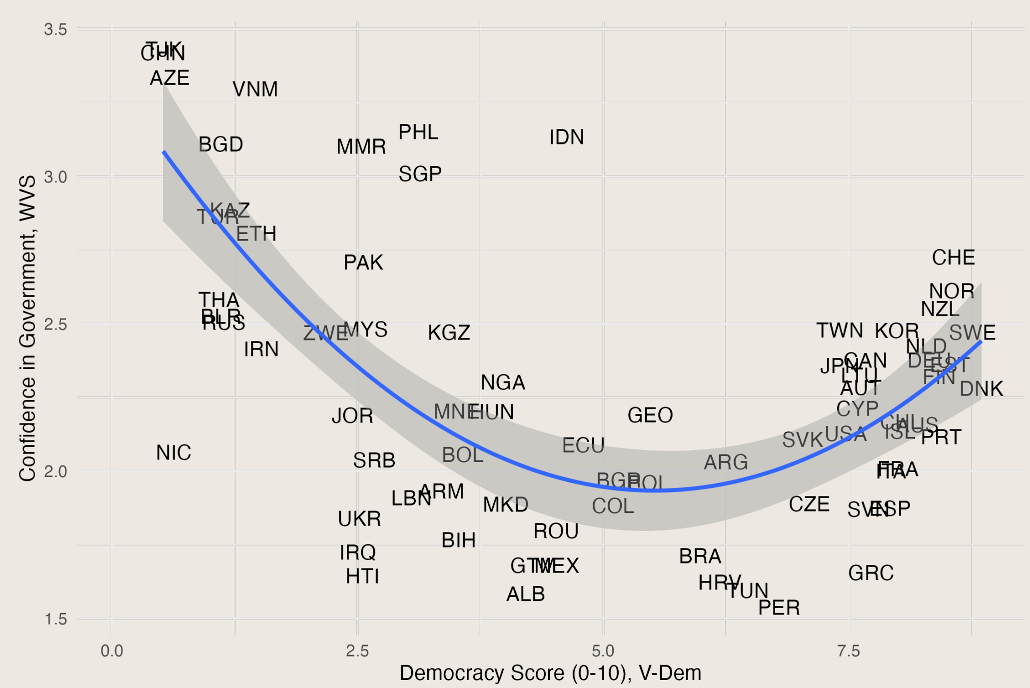

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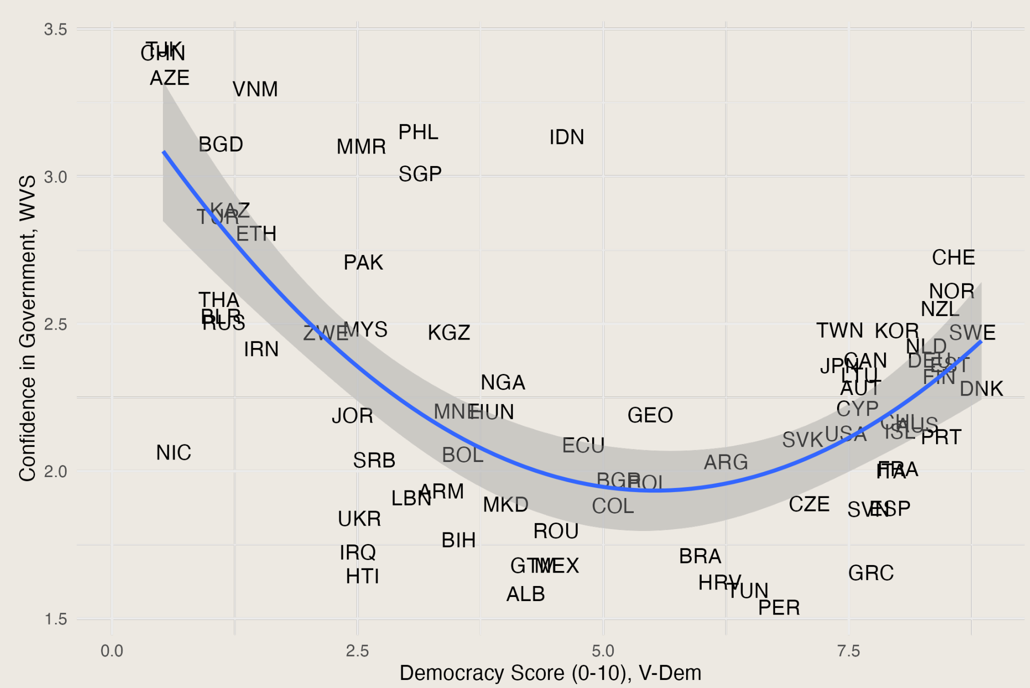

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- **Vertex:** $-\beta_1/(2\beta_2)$. This is where sign of the relationship changes — **may fall outside the observed range of** X.

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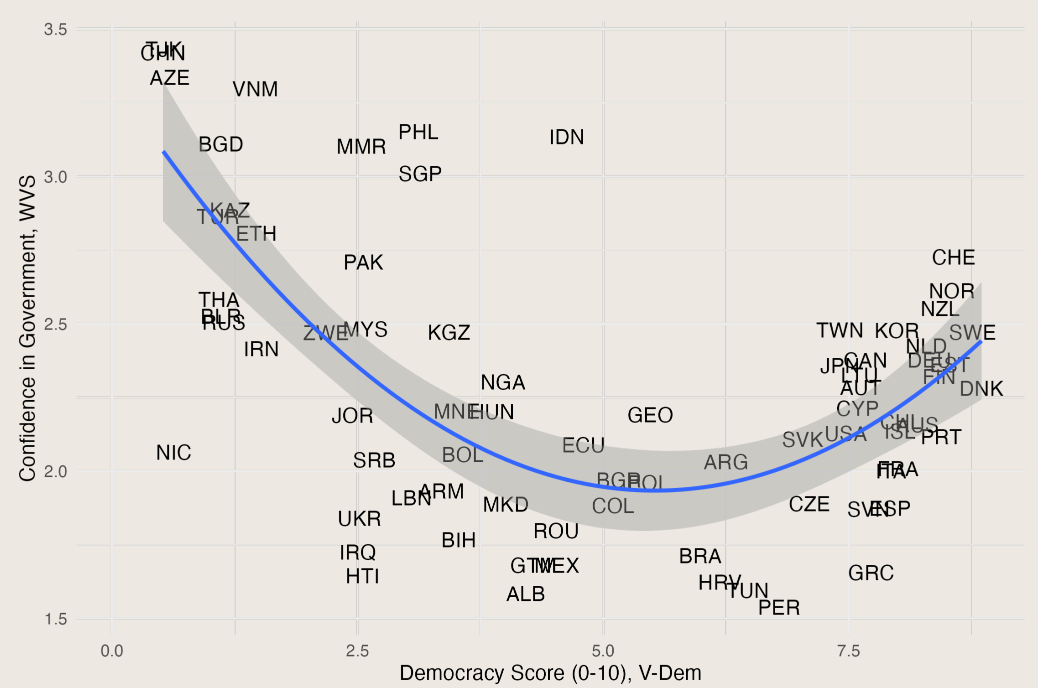

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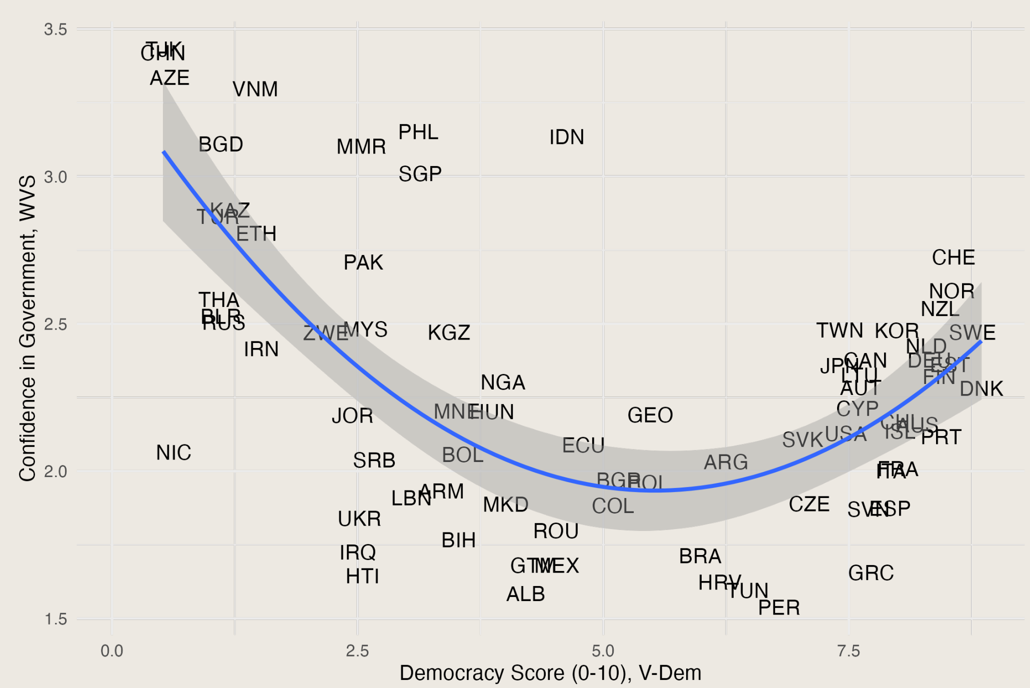

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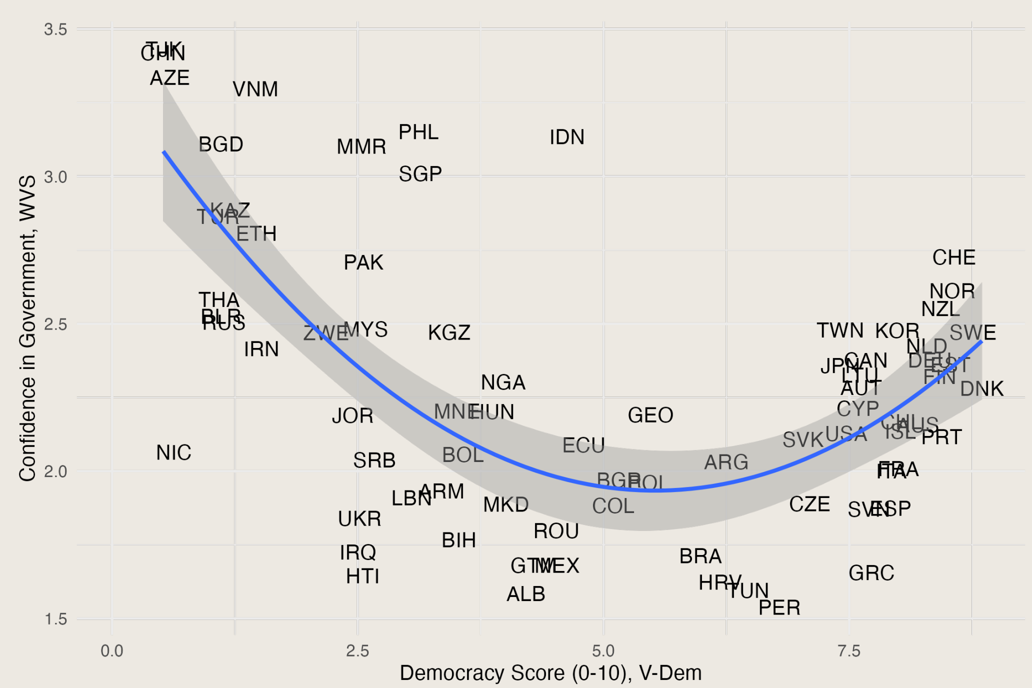

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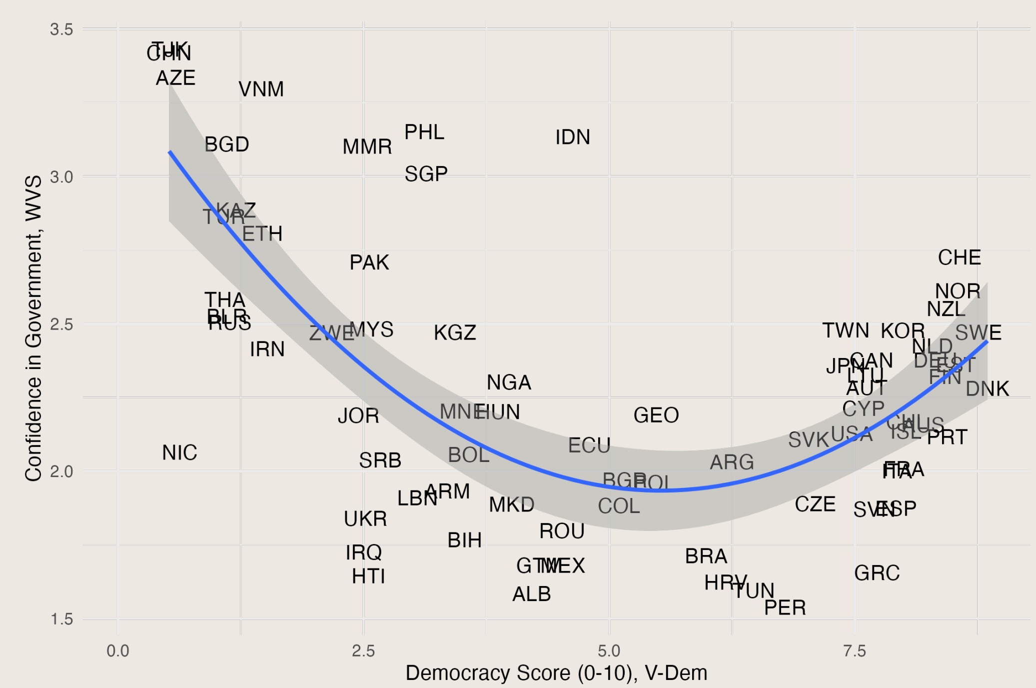

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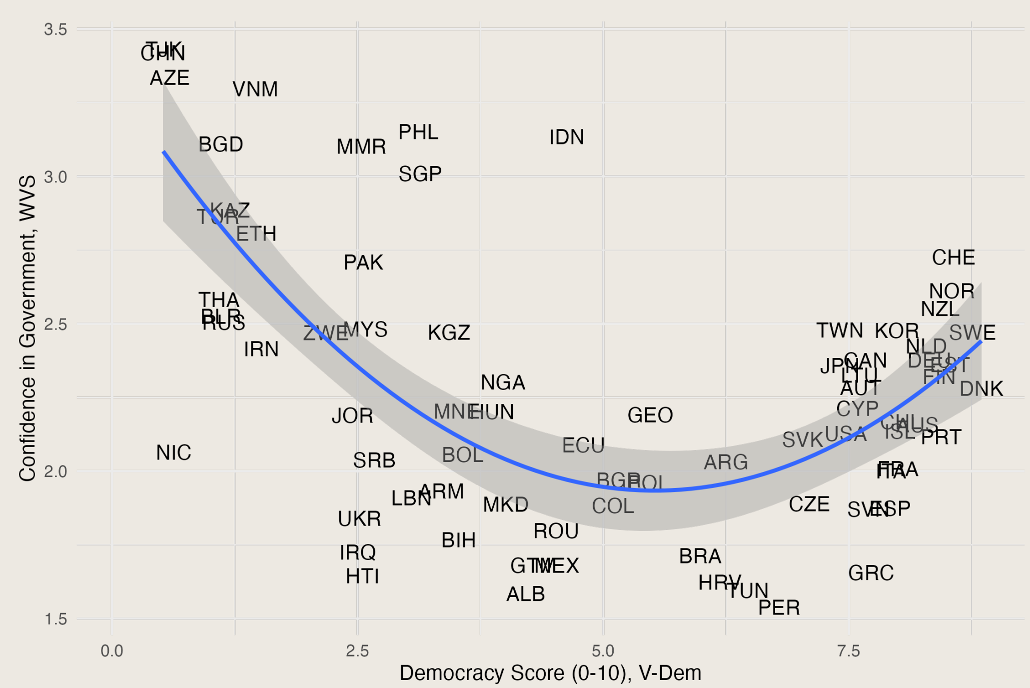

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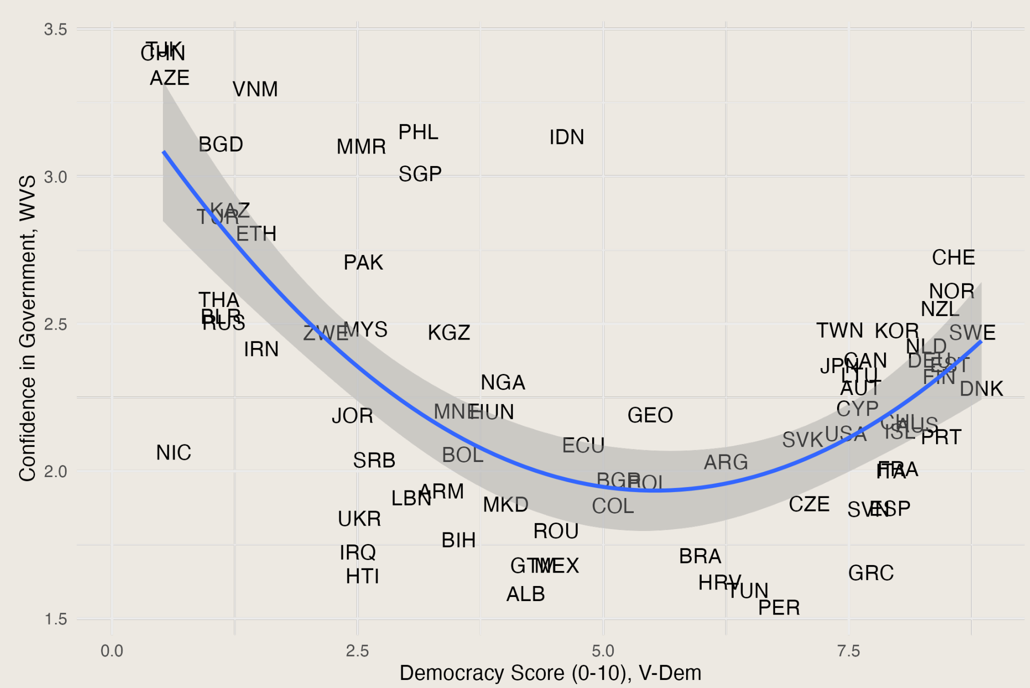

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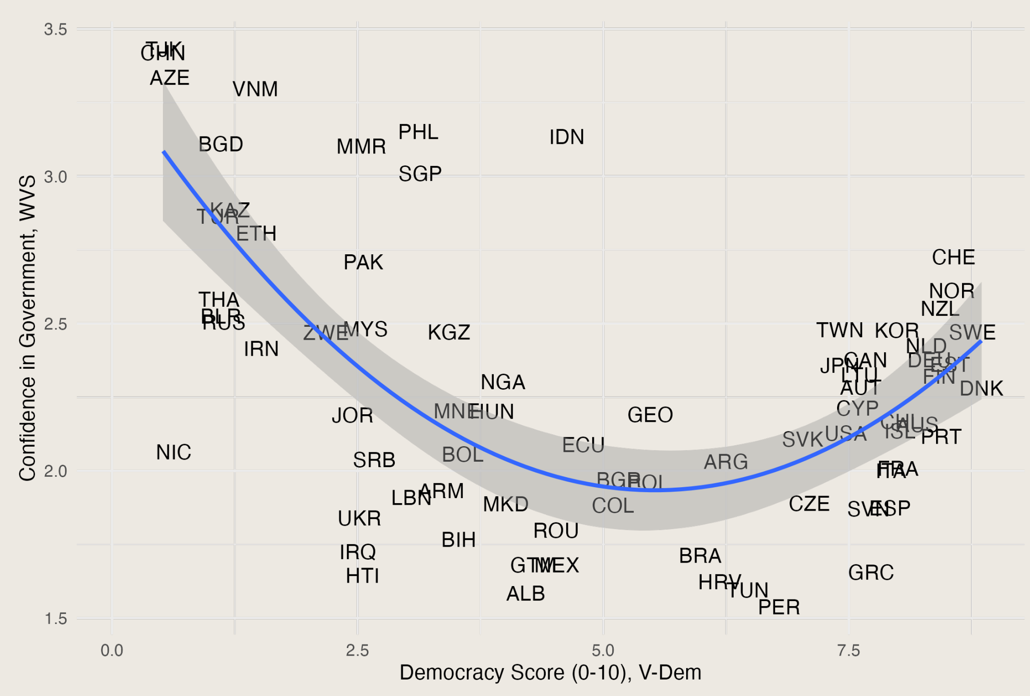

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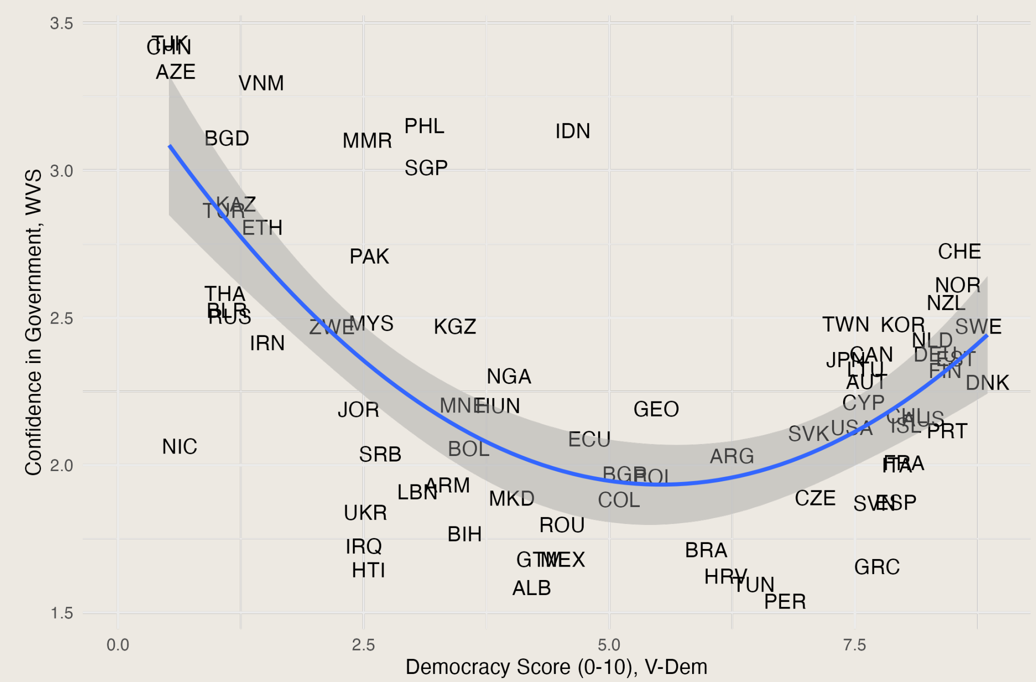

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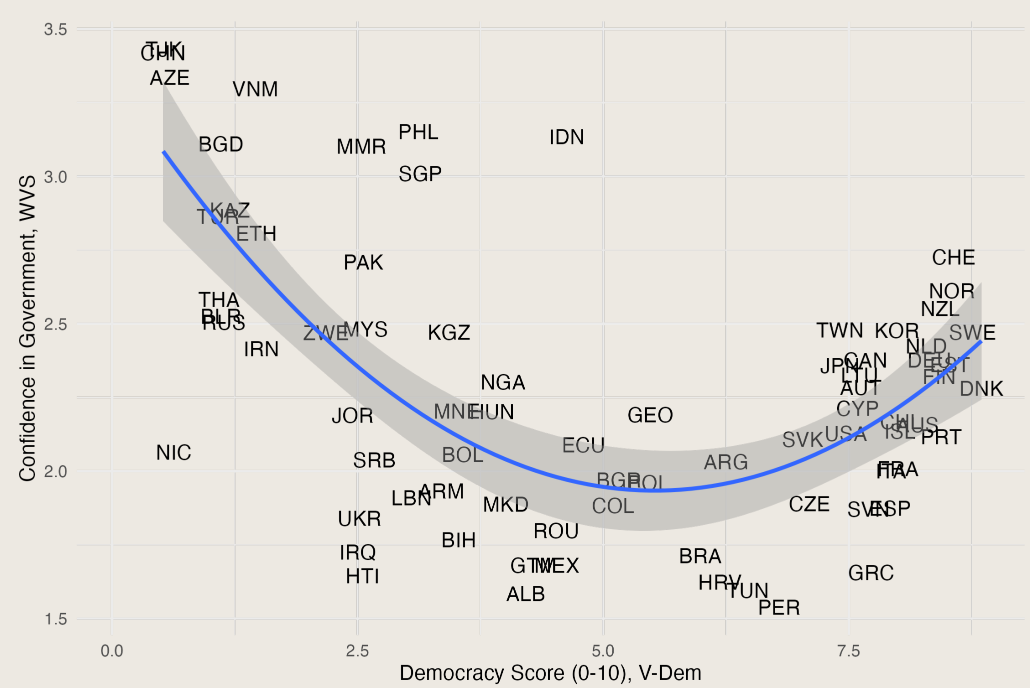

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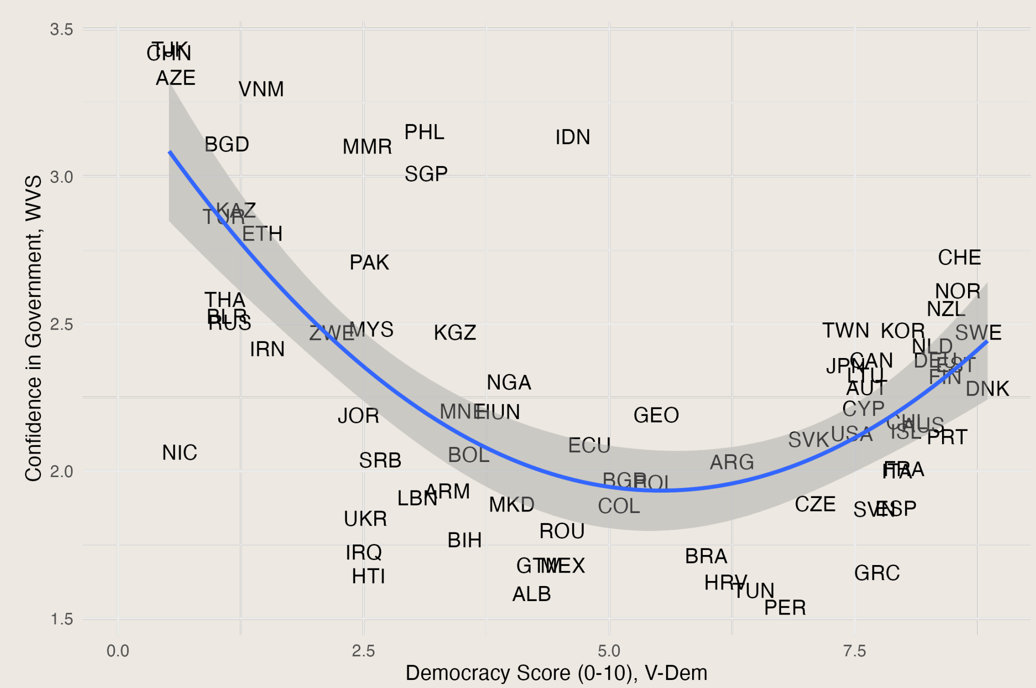

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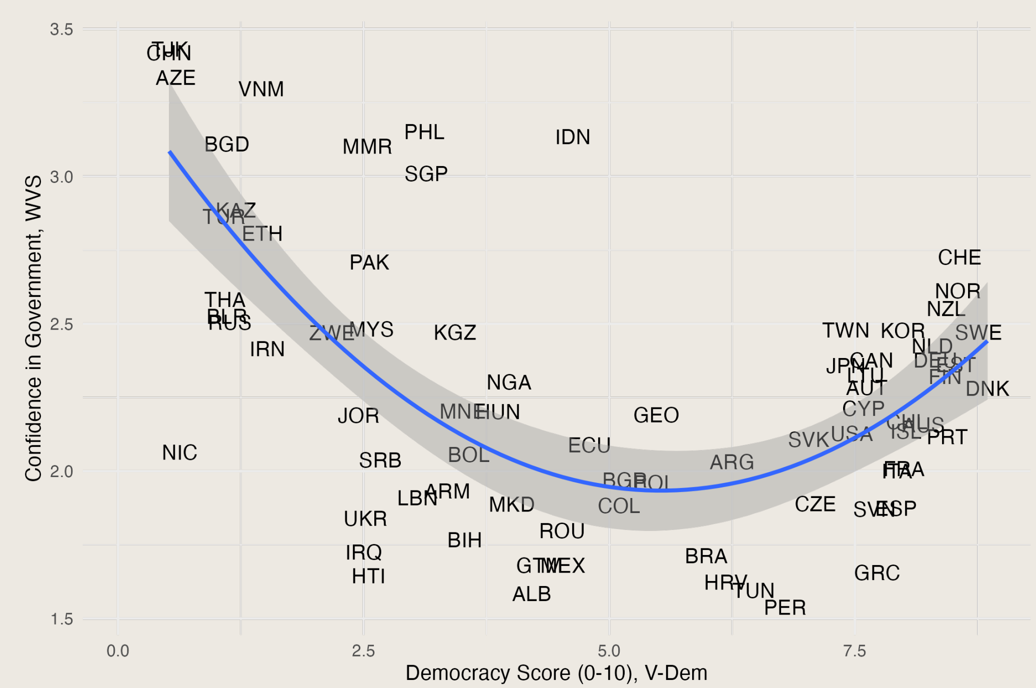

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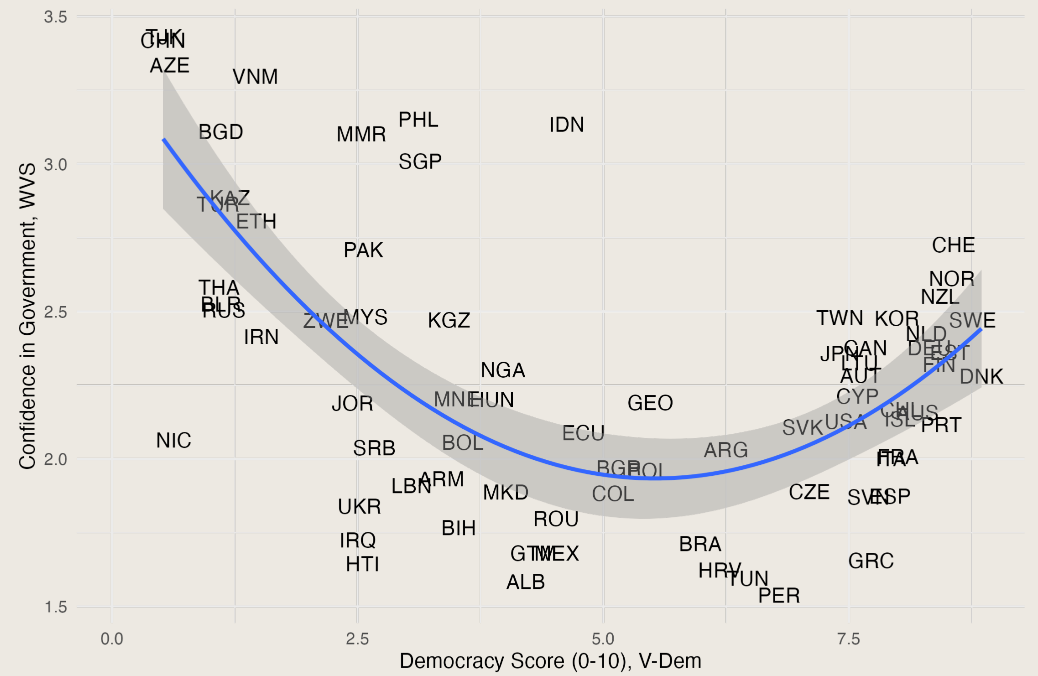

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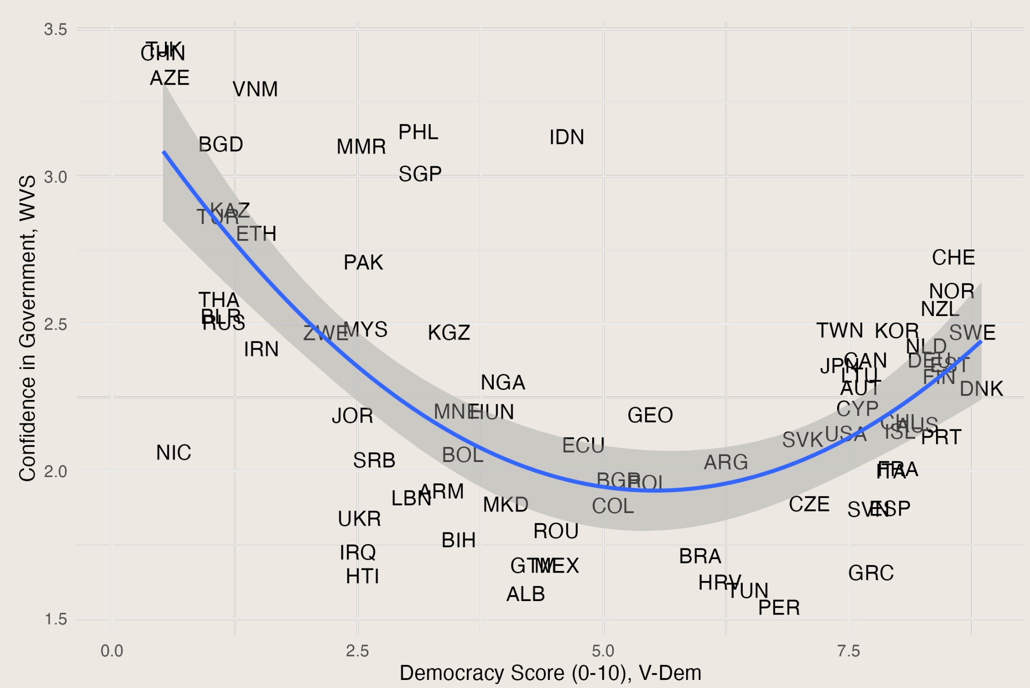

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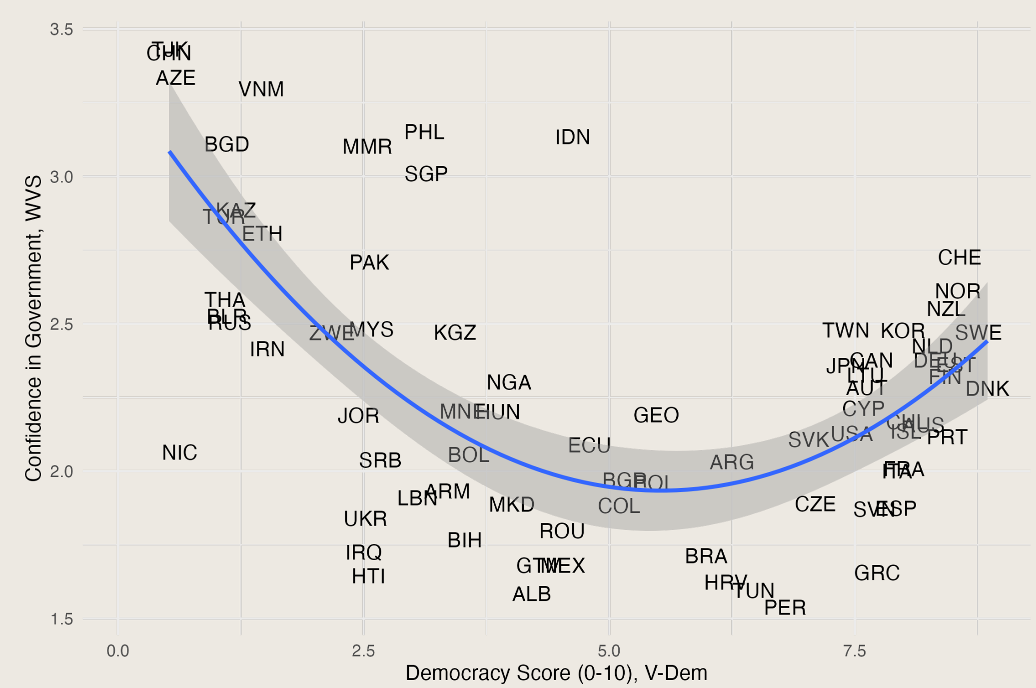

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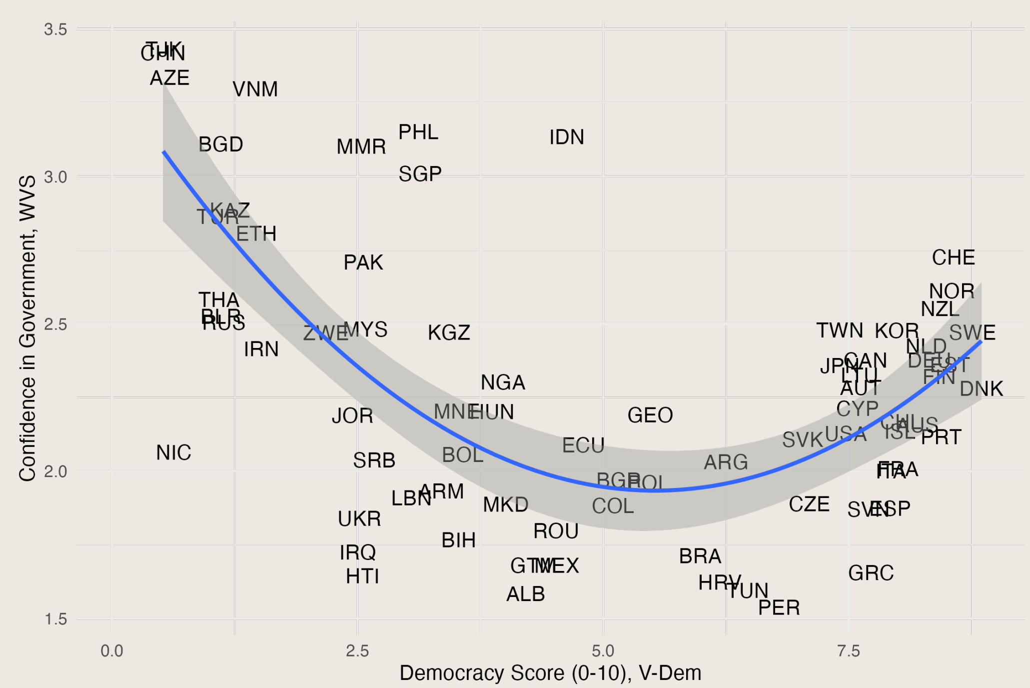
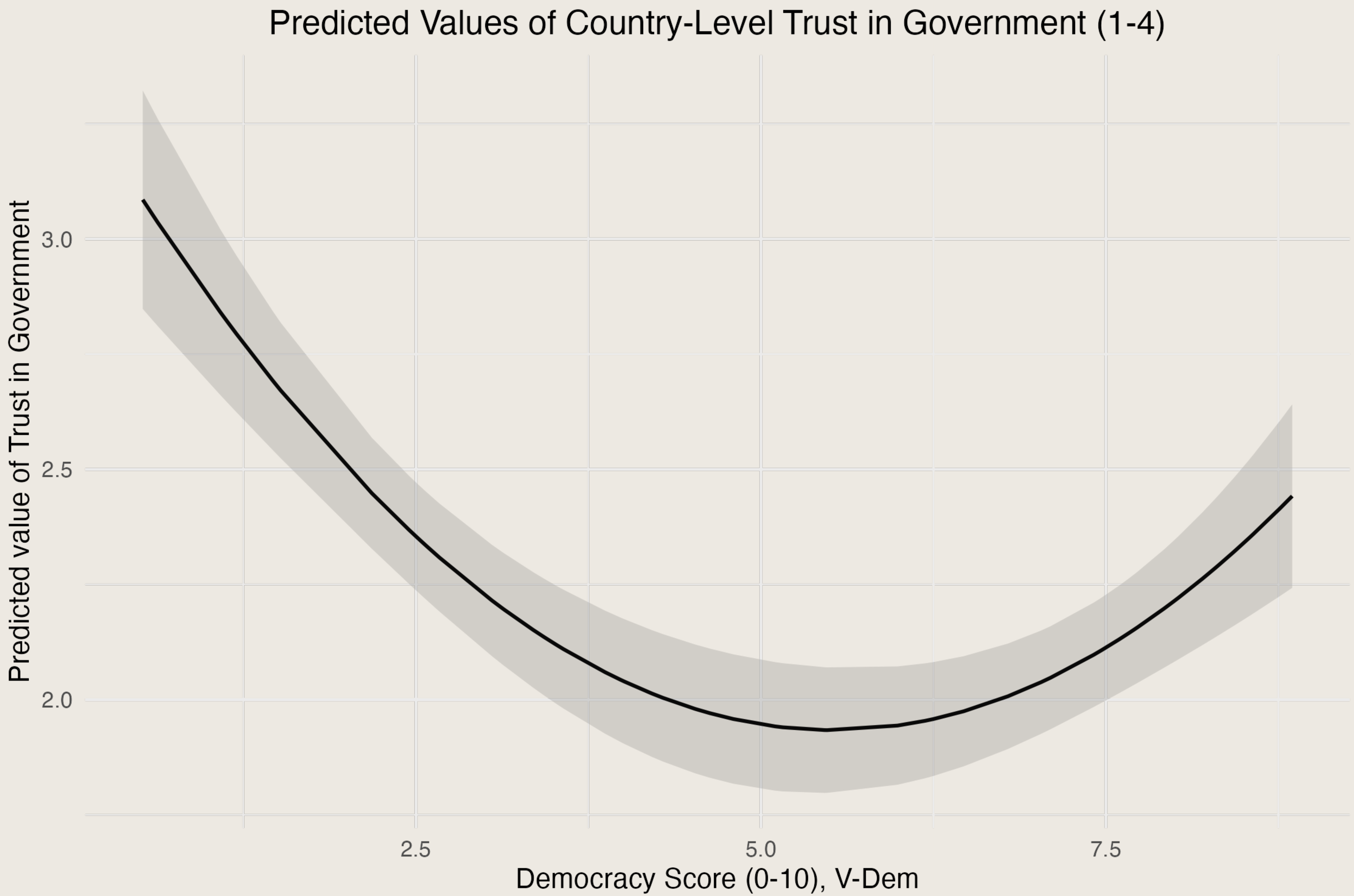


Polynomial Terms in R

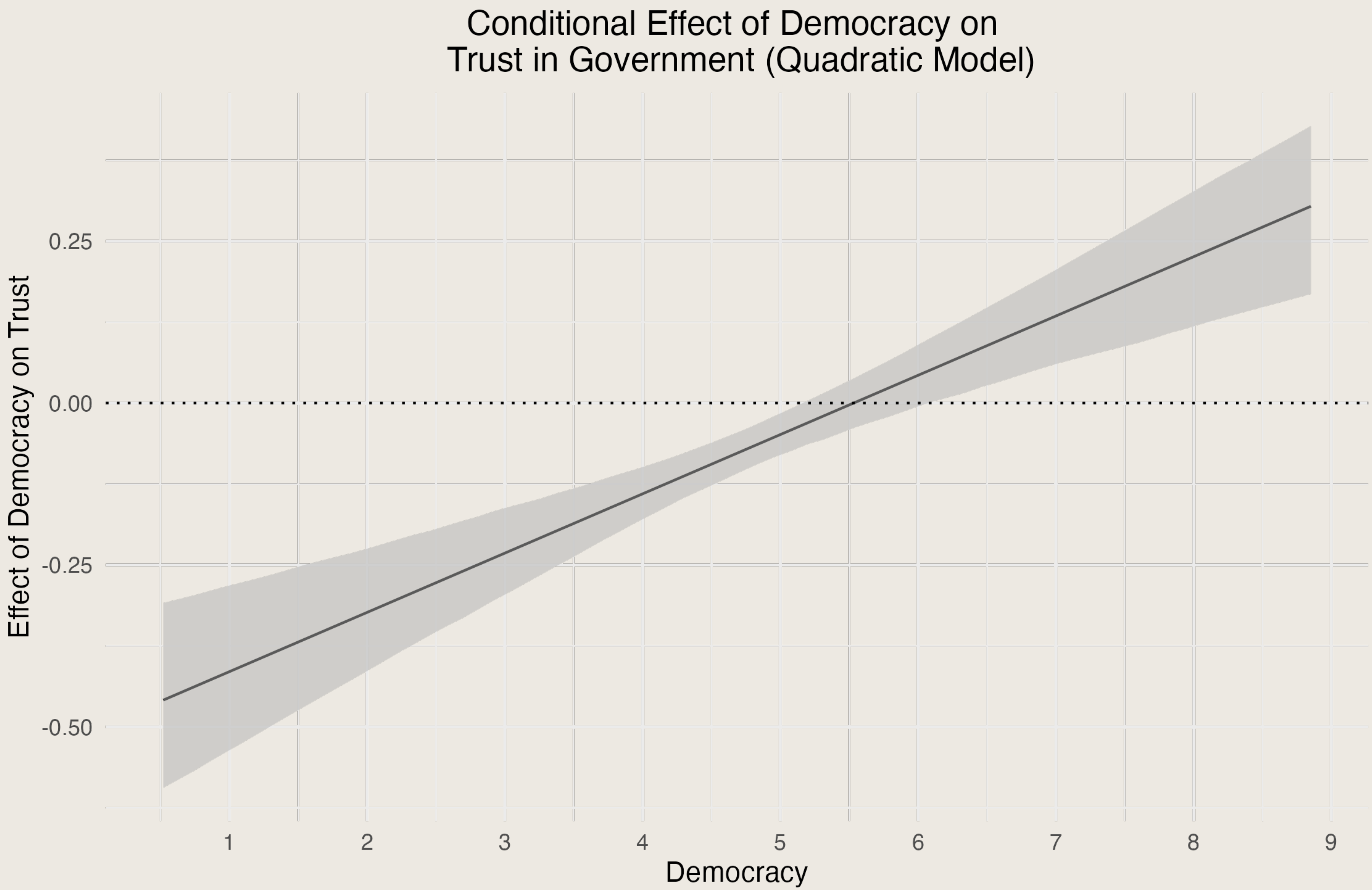
```
> model1 <- lm(conf_goverment ~ democracy + I(democracy^2), data = qog)
> stargazer(model1, type = "text", single.row = TRUE)

===============================================
                       Dependent variable:
                   ----------------------------
                          conf_goverment
-----------------------------------------------
democracy                -0.508*** (0.076)
I(democracy2)            0.046*** (0.008)
Constant                 3.337*** (0.152)
-----------------------------------------------
Observations                    76
R2                             0.417
Adjusted R2                    0.401
Residual Std. Error        0.366 (df = 73)
F Statistic           26.076*** (df = 2; 73)
===============================================
Note:               *p<0.1; **p<0.05; ***p<0.01
```

Visualisation: Predicted Values Plot



Visualisation: Conditional Effect Plot



Check if you understand

- How does a leader's time in office affect spending in Chinese counties?

Dependent Variable: Annual Growth Rate of Expenditures Per Capita Explanatory Variables	Party Secretary Model Coefficient (Standard Error)	
$(\text{Time in office})^2$	−0.3946**	−0.4860**
	(0.1728)	(0.2049)
Time in office	2.4793**	3.1624**
	(1.0212)	(1.2252)
Annual growth rate of revenues per capita	0.2493***	0.2589***
	(0.0142)	(0.0166)
Annual growth rate of subsidies per capita		0.1411***
		(0.0092)

- Guo, G. (2009). China's local political budget cycles. *American Journal of Political Science, 53*(3), 621-632.

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- Interpretation gets trickier. Use visualisation tools to get a sense of what you're fitting.

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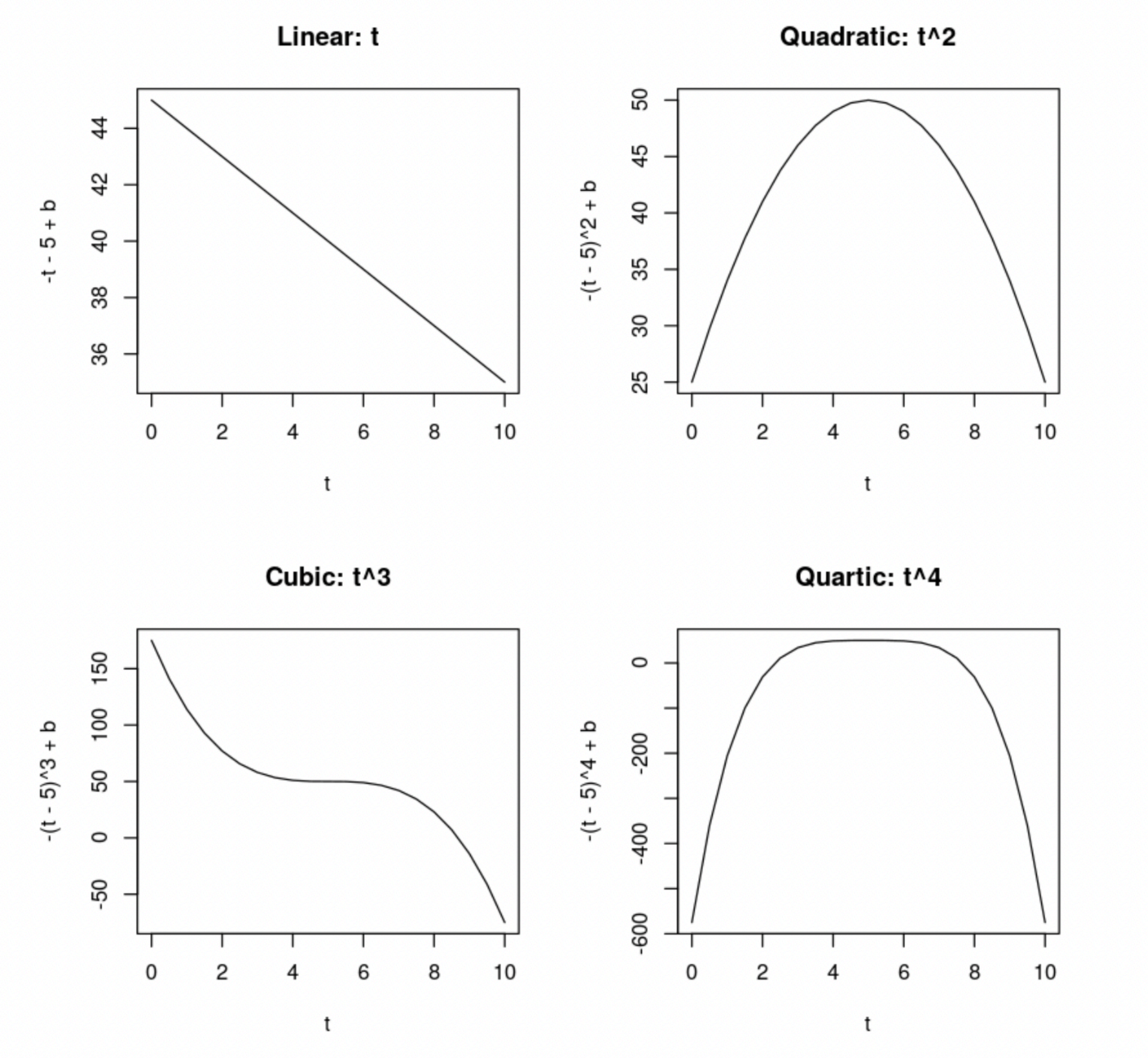

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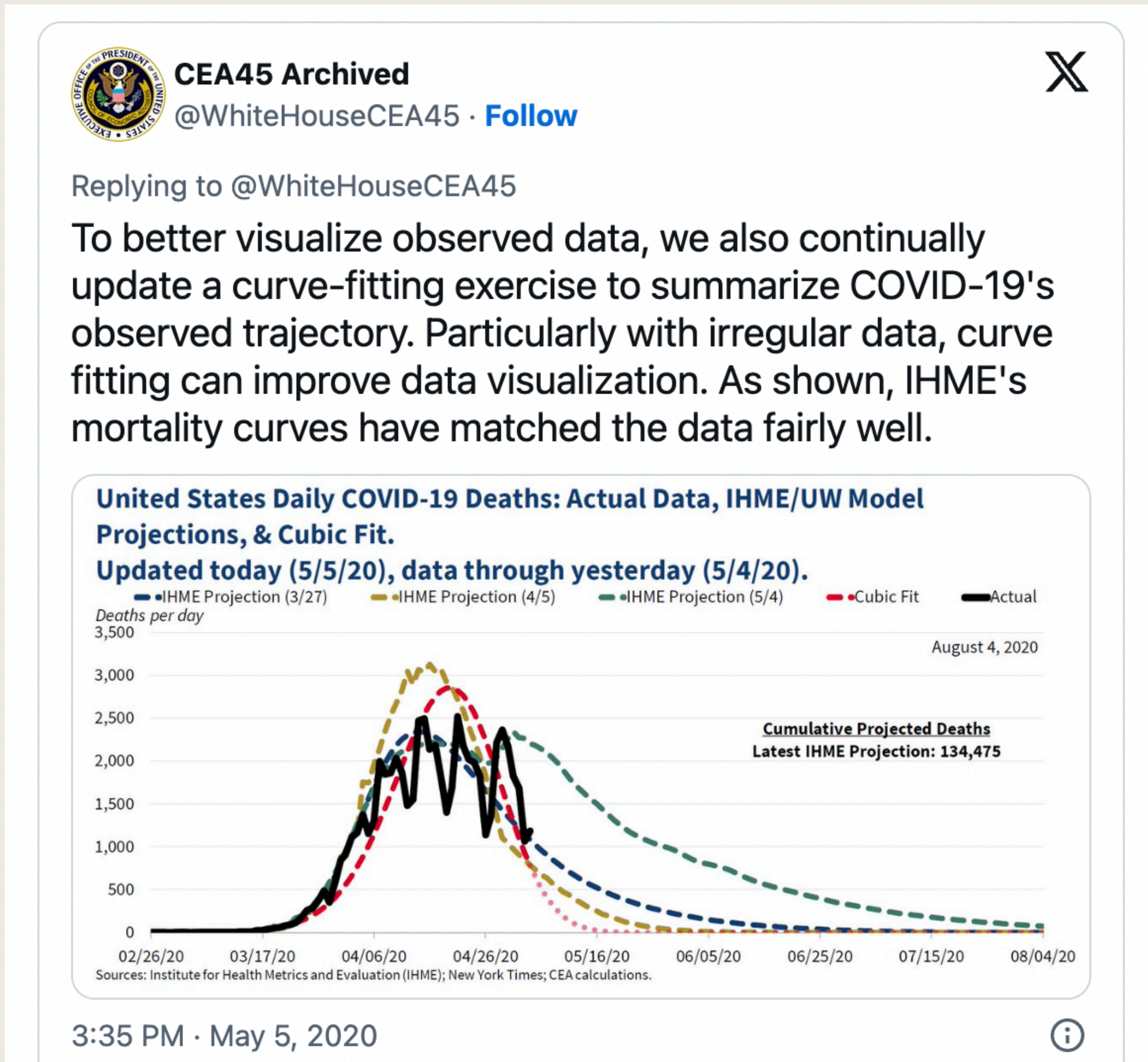

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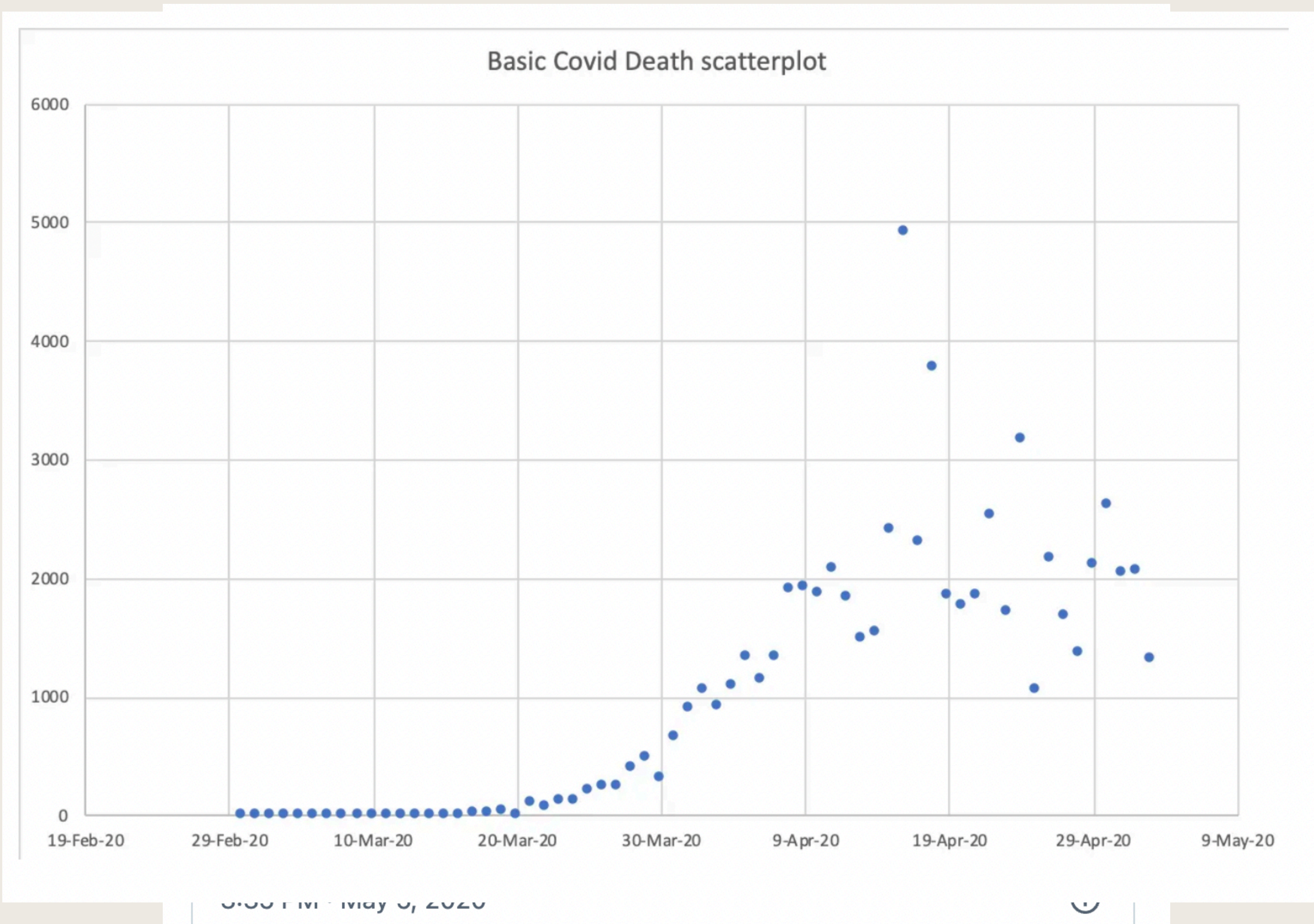

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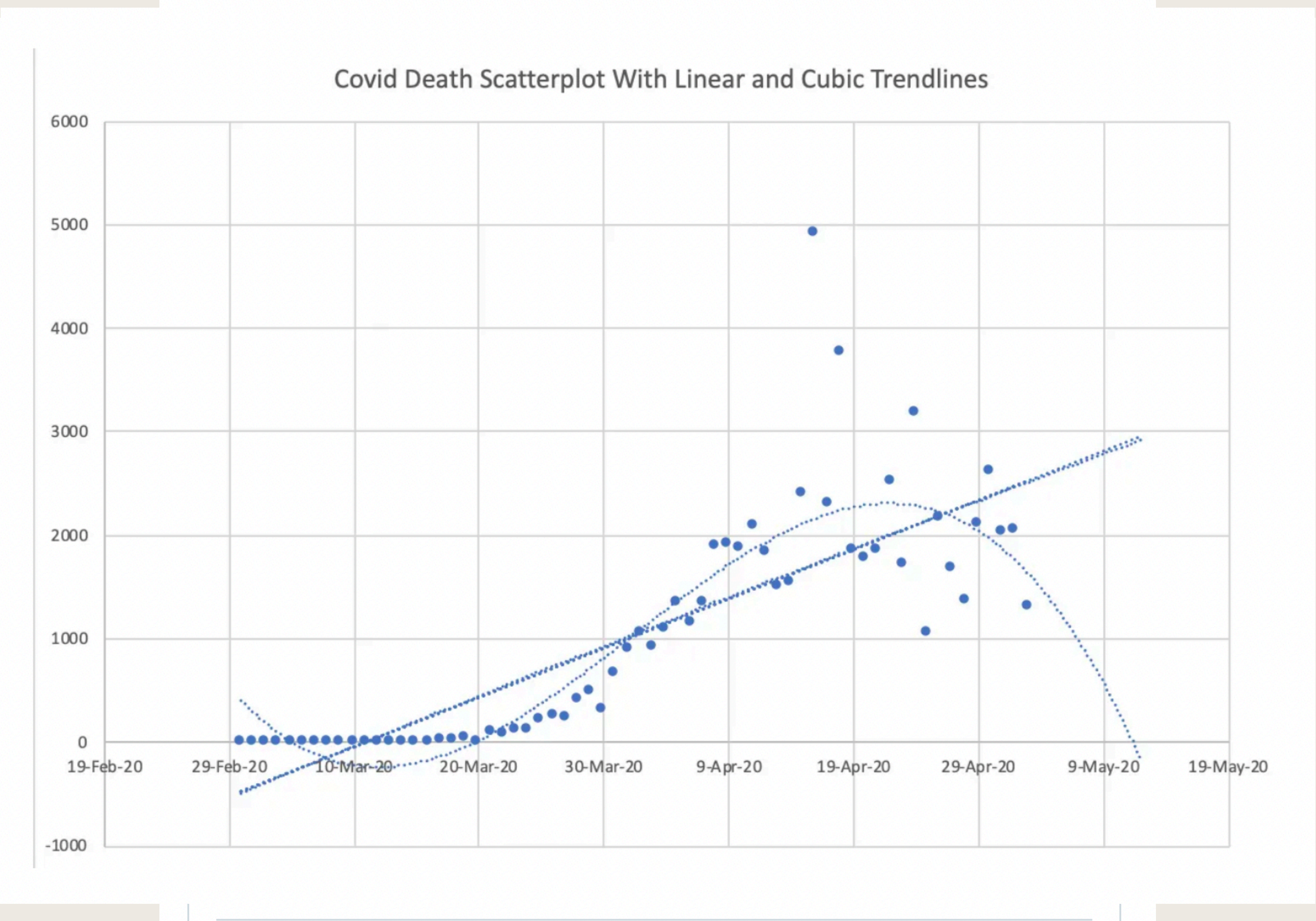

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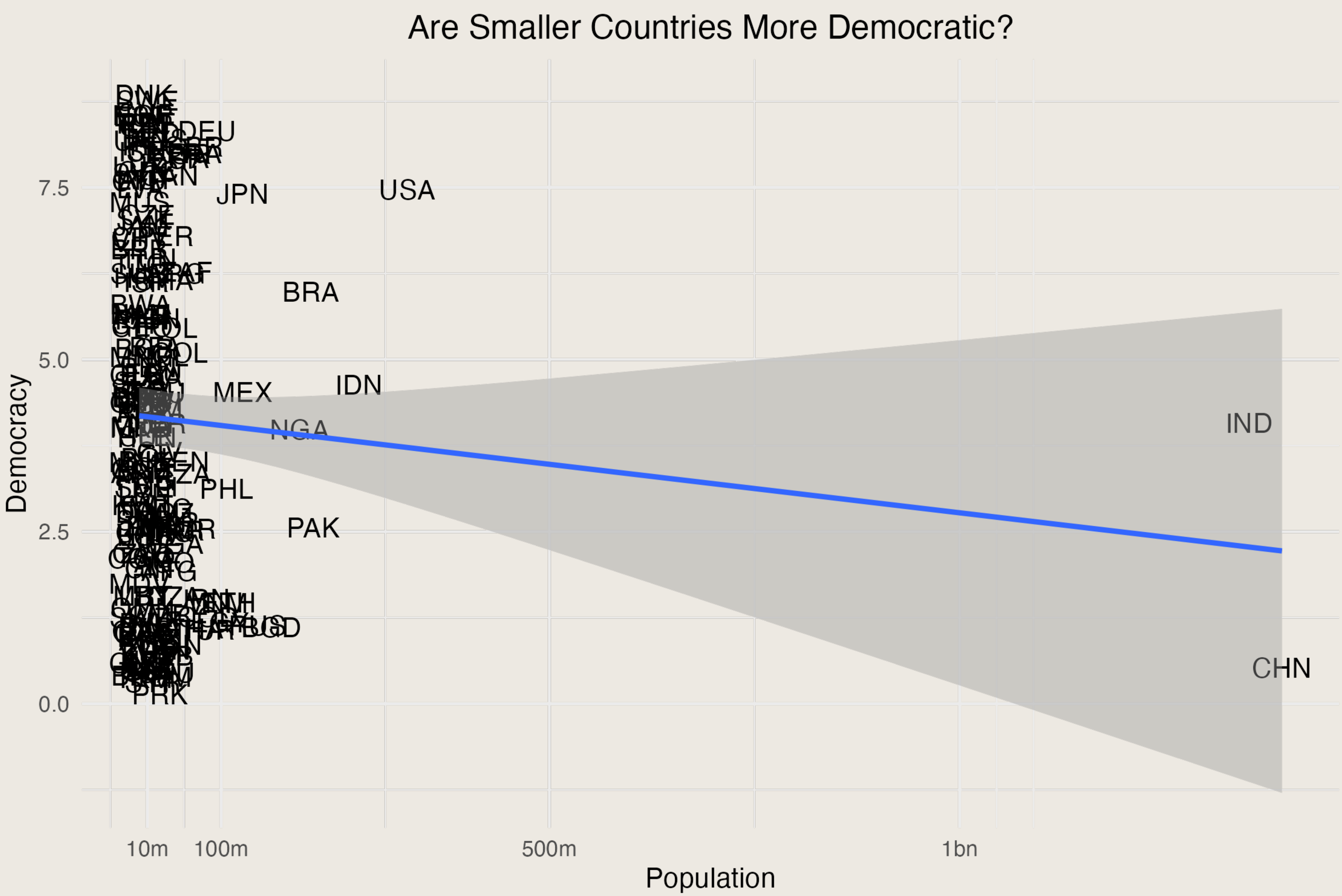

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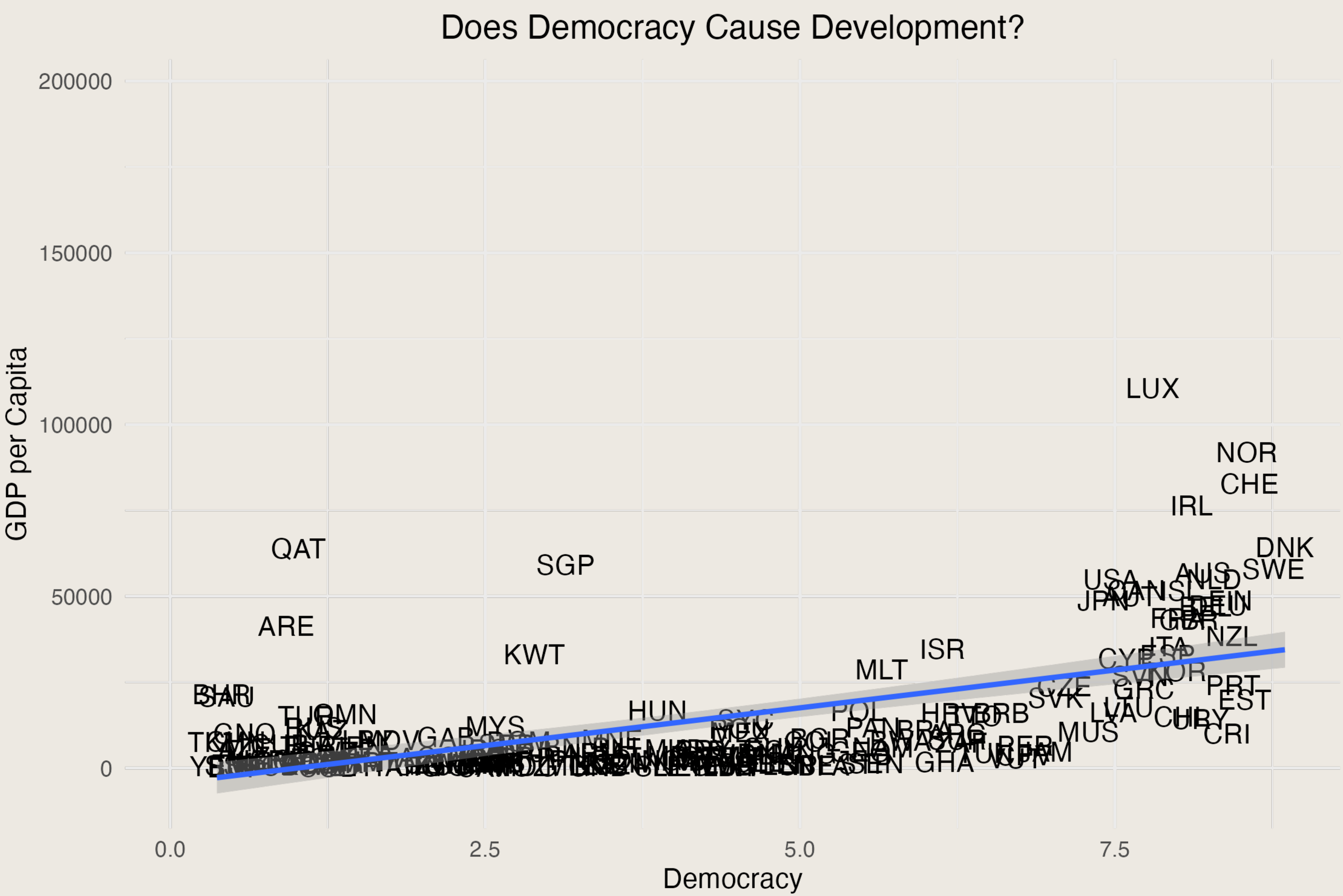

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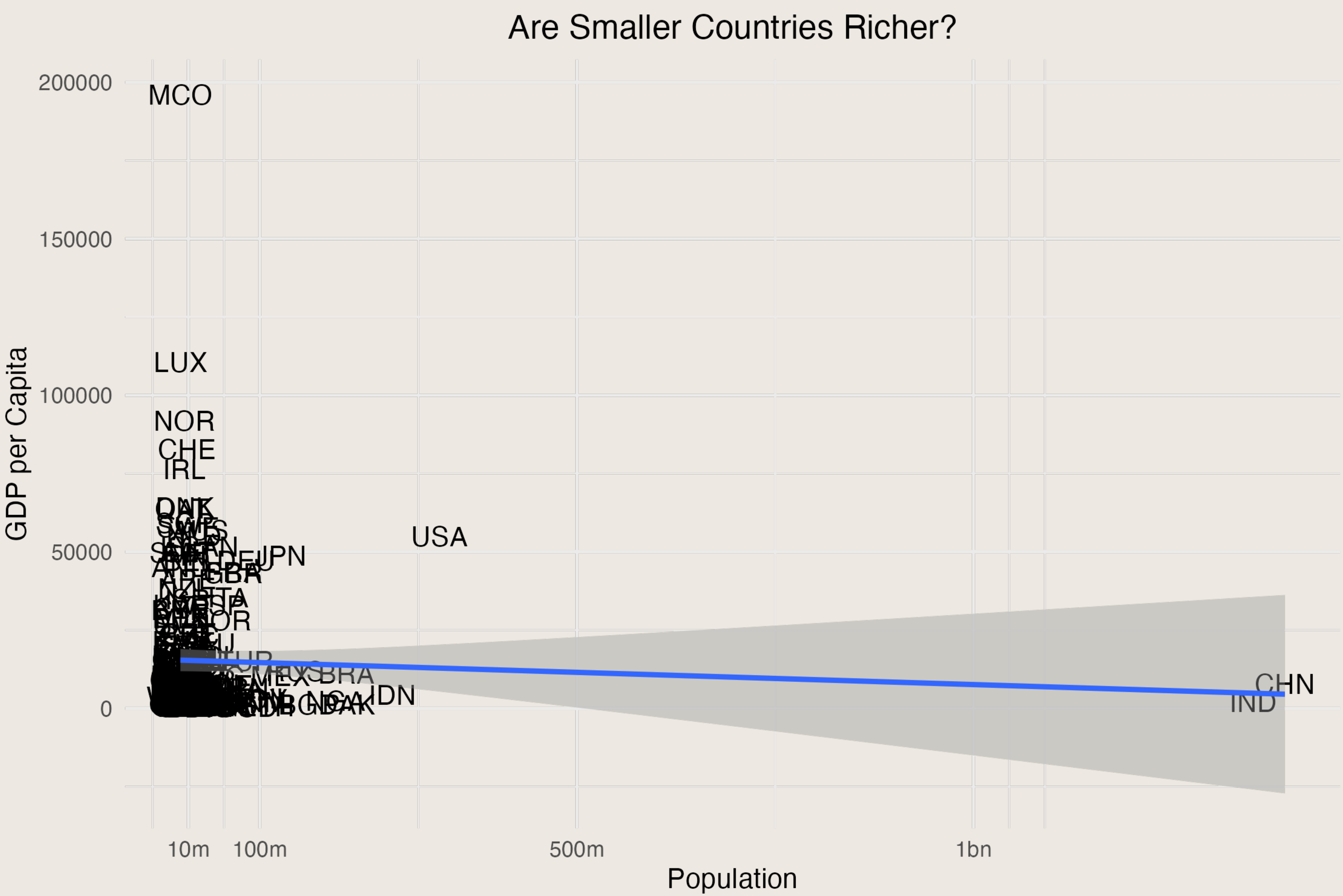

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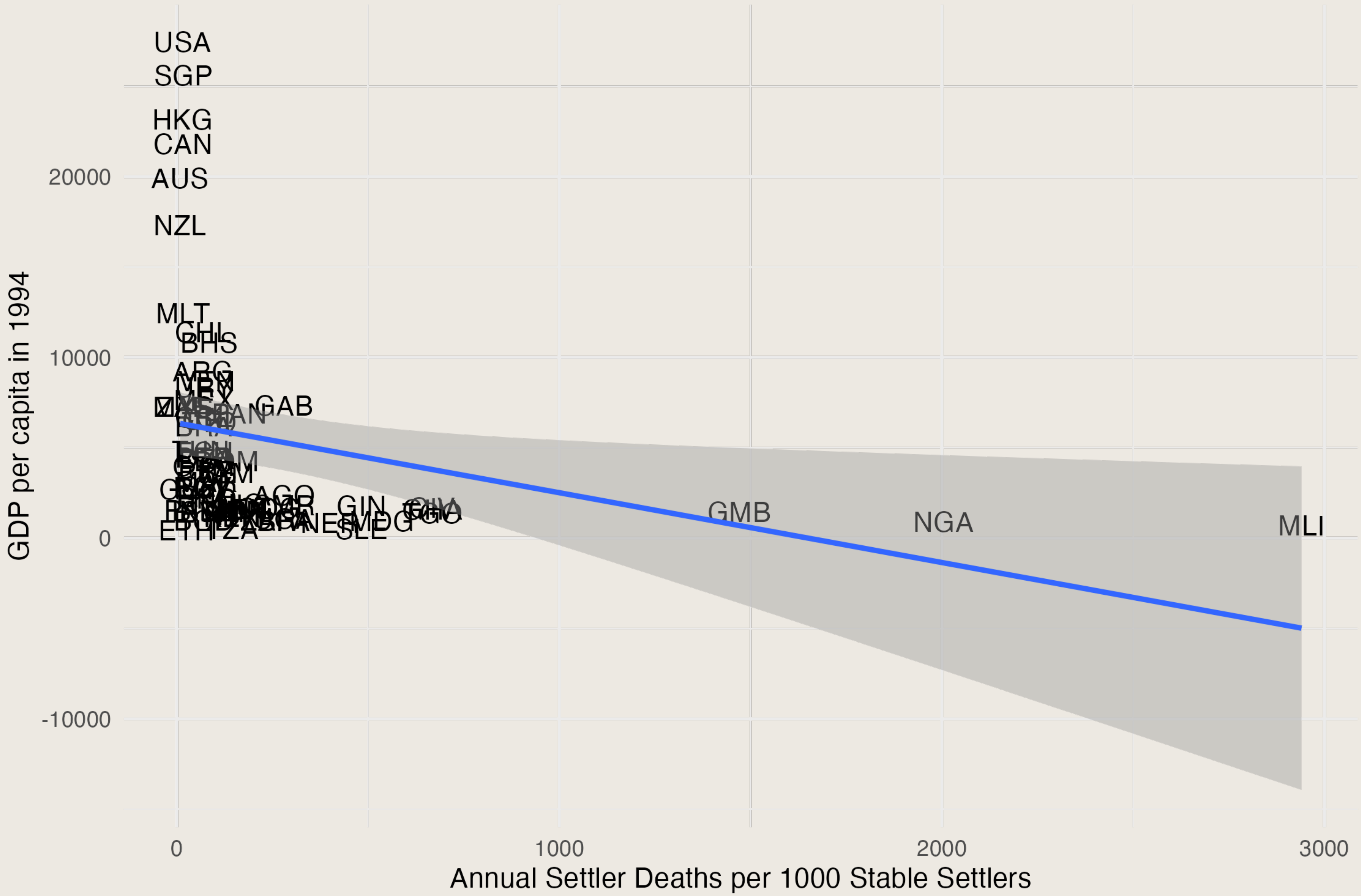

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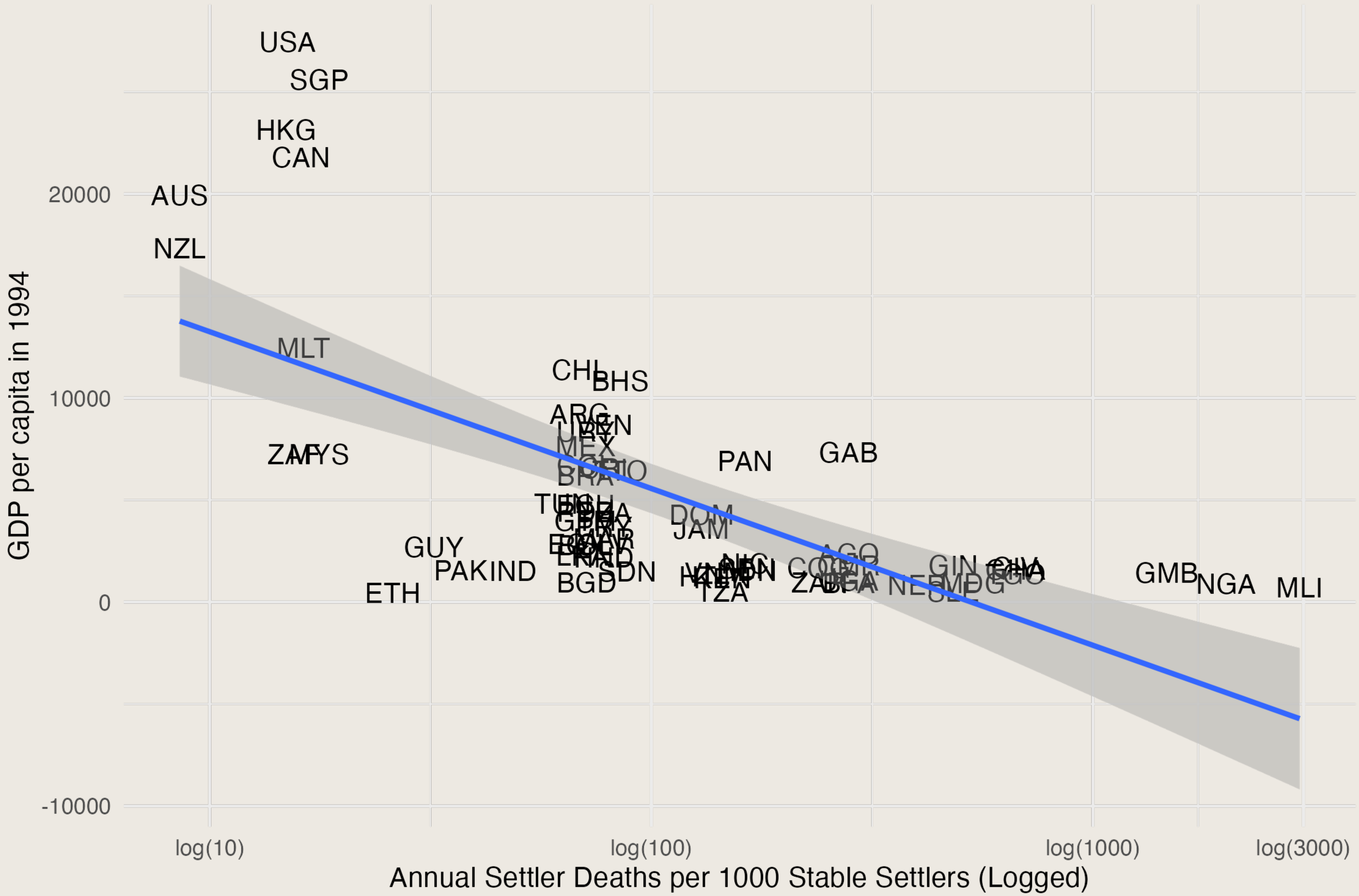

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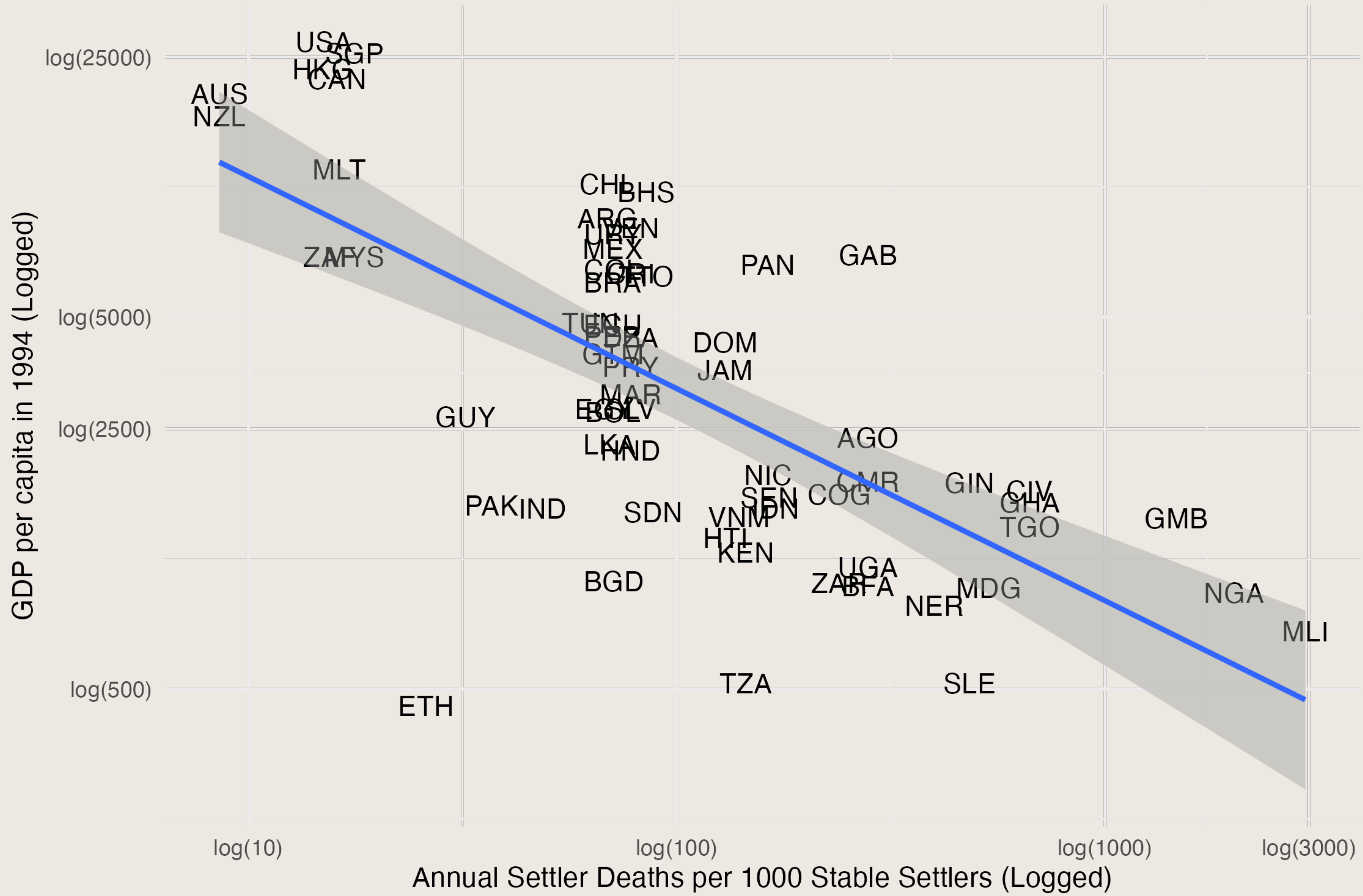

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Log Coefficients: Interpretation

	Dependent variable:			
	GDP Per Capita		log(GDP per Capita)	
	(1)	(2)	(3)	(4)
Settler Mortality	-3.862**		-0.001***	
	(1.637)		(0.0003)	
log(Settler Mortality)		-3,336.467***		-0.570***
		(485.995)		(0.078)
Constant	6,374.983***	20,929.100***	8.275***	10.700***
	(866.715)	(2,337.663)	(0.136)	(0.374)
Observations	64	64	64	64
R2	0.082	0.432	0.169	0.464
Adjusted R2	0.068	0.423	0.156	0.456
Model Type	**Level-Level**	**Level-Log**	**Log-Level**	**Log-Log**
Note:		*p<0.1;	**p<0.05;	***p<0.01

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  * Non-linearities produced by skewed, positive variables.
  * **Assume proportional relationships**: halving X has approximately the same effect size on Y as doubling X.

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* Method options are sprawling and changing fast (AI is coming for all of us) — make your methods training fit your research needs, not the other way around.

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- Long-term investment will involve some self-learning.

How did you like the course?

DPIR MT24 Course Content &
Teaching Feedback NEW!

Thank you for your kind attention!

Leonardo Carella

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